Basic Mathematics and Logarithm

EXERCISES

ELEMENTARY

Q.1 (3)

A & B are two rational number then $\frac{A}{B}$ is

Also rational number if $B \neq 0$.

Q.2 (4)

It is a property

Q.3 (4)

$\underset{x}{(\text{rational})} \times \underset{y}{(\text{irrational})}$ = irrational except when x = 0

Q.4 (1)

Every irrational number can be expressed on the number line. This statement is always true.

Q.5 (1)

Here $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$

Here sum of three + ve number can not be equal to zero then all three number independently be zero.

$\therefore$ There are no any real root.

Q.6 (2)

$a(a-b) + b(b-c) + c(c-a) = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

Multiplying & deviding by 2,

$\frac{1}{2}[(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ac)] = 0$

$\Rightarrow \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \Rightarrow a = b = c$

Q.7 (1)

$2x^3 - 5x^2 + x + 2$

$= ax^3 + (-b - 2a)x^2 + (a - 1 + 2b)x + 2$

On comparing coefficient of x^3 & x, we get

a = 2 & b = 1

Q.8 **(3)**

Putting x = 1, remainder = 7

Q.9 (3)

Let $P(x) = x^3 - a^2x + x + 2$ be the given polynomial

Then by factor theorem, (x – a) is a factor of P(x) iff P(a) = 0

$\Rightarrow a^3 - a^2 . a + a + 2 = 0$

$\Rightarrow a + 2 = 0 \Rightarrow a = -2$

Q.10 **(1)**

$$x^{x.x^{1/3}} = \left(x.x^{1/3}\right)^x \Rightarrow x^{x^{1+\frac{1}{3}}} = \left(x^{1+\frac{1}{3}}\right)^x$$

$$\Rightarrow x^{x^{4/3}} = \left(x^{4/3}\right)^x = x^{x^{4/3}} = x^{\frac{4}{3}x} \Rightarrow x^{4/3} = \frac{4}{3}x;$$

Also is an obvious solution.

Q.11 **(2)**

$\log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8 . \log_8 9 =$

$\frac{\log 4}{\log 3} . \frac{\log 5}{\log 4} . \frac{\log 6}{\log 5} . \frac{\log 7}{\log 6} . \frac{\log 8}{\log 7} . \frac{\log 9}{\log 8} = \frac{\log 9}{\log 3}$

$= \log_3 9 = \log_3 3^2 = 2.$

Q.12 **(3)**

$$a = \log_{24} 12 = \frac{\log 12}{\log 24} = \frac{2\log 2 + \log 3}{3\log 2 + \log 3}$$

$$b = \log_{36} 24 = \frac{3\log 2 + \log 3}{2(\log 2 + \log 3)}$$

$$c = \log_{48} 36 = \frac{2(\log 2 + \log 3)}{4\log 2 + \log 3}$$

$$\therefore abc = \frac{2\log 2 + \log 3}{4\log 2 + \log 3}$$

$$\Rightarrow 1 + abc = \frac{6\log 2 + 2\log 3}{4\log 2 + \log 3} = 2.\frac{3\log 2 + \log 3}{4\log 2 + \log 3} = 2bc.$$

Q.13 (2)

Applying base change theorem,

$= \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab}$

$= \log_{abc}\sqrt{bc} . \sqrt{ca} . \sqrt{ab} = \log_{abc} abc = 1$

Q.14 (3)

$\log_2 15 . \log_{1/6} 2 . \log_3 1/6$

$$= \frac{\log_e 15}{\log_e 2} \times \frac{\log_e 2}{\log_e 1/6} \times \frac{\log_e 1/6}{\log_e 3} = \frac{\log_e 15}{\log_e 3}$$

$$= \frac{\log_e (3\times 5)}{\log_e 3} = 1 + \log_3 5$$

$\therefore [1 + \log_3 5] = 2$

Q.15 **(2)**

$x = \log_a bc \Rightarrow 1 + x = \log_a a + \log_a bc = \log_a abc$

$\therefore (1 + x)^{-1} = \log_{abc} a$

$\therefore (1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1} = \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1.$

Q.16 **(4)**

$$\log_{1000} x^2 = \log_{10^3} x^2 = 2\log_{10^3} x = \frac{2}{3}\log_{10} x = \frac{2}{3}y$$

Q.17 **(1)**

Let *x* be the required logarithm , then by definition

$$\log_{2\sqrt{2}} 32\sqrt[5]{4} = x$$

$$\left(2\sqrt{2}\right)^x = 32\sqrt[5]{4} \Rightarrow \left(2.2^{1/2}\right)^x = 2^5.2^{2/5} \; ; \; \therefore \; 2^{\frac{3x}{2}} = 2^{5+\frac{2}{5}}$$

Here, by equating the indices, $\frac{3}{2}x = \frac{27}{5}$

$\therefore x = \frac{18}{5} = 3.6$.

Q.18 **(1)**

$$\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log_e a + \log_e b\right)$$

$$= \frac{1}{2}\log_e(ab) = \log_e\sqrt{ab}$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab} \Rightarrow a+b = 2\sqrt{ab}$$

$$\Rightarrow \left(\sqrt{a}-\sqrt{b}\right)^2 = 0 \Rightarrow \sqrt{a}-\sqrt{b} = 0 \Rightarrow a = b$$

Q.19 **(3)**

$$\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = \log_7 \log_7 7^{7/8} = \log_7(7/8)$$

$= \log_7 7 - \log_7 8 = 1 - \log_7 2^3 = 1 - 3\log_7 2.$

Q.20 **(4)**

$$81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$$

$$= 3^{4\log_3 5} + 3^{3.\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$$

$$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$$

$$= 5^4 + 36^{3/2} + 7^2 = 890$$

Q.21 **(3)**

$$\log_7 \log_5\left(\sqrt{x^2+5+x}\right) = 0 = \log_7 1$$

$$\Rightarrow \log_5\left(x^2+5+x\right)^{1/2} = 1 = \log_5 5$$

$\Rightarrow (x^2 + 5 + x)^{1/2} = 5$

$\Rightarrow (x^2 + x + 5) = 25 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x - 4)(x + 5) = 0 \Rightarrow x = 4, -5 \Rightarrow x = 4$

Q.22 **(3)**

$y = 3^{12} \times 2^8 \Rightarrow \log_{10}y = 12\log_{10}3 + 8\log_{10}2$

$= 12 \times 0.47712 + 8 \times 0.30103$

$= 5.72544 + 2.40824 = 8.13368$

$\therefore$ Number of digits in y = 8 + 1 = 9.

Q.23 (4)

$\log_{1-x}(x - 2) \geq -1$

$1 - x > 0 \Rightarrow 1 > x \Rightarrow x \in (-\infty, 1) - \{0\}$

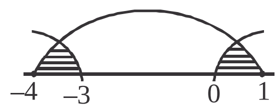
$x - 2 > 0 \Rightarrow x > 2$ No solution.

Q.24 (2)

$2 - \log_2(x^2 + 3x) \geq 0$

$\log_2(x^2 + 3x) \leq 2$

$\Rightarrow x^2 + 3x \leq 4$



$\Rightarrow x^2 + 3x - 4 \leq 0$

$\Rightarrow (x + 4)(x - 1) \leq 0$

$\Rightarrow x \in [-4, 1]$

and $x^2 + 3x > 0 \Rightarrow x \in (-\infty, -3) \cup (0, \infty)$

Ans. : $[-4, -3) \cup (0, 1]$

Q.25 (2)

$$\log_{\sqrt{0.9}} \log_5\left(\sqrt{x^2+5+x}\right) > 0$$

$$\log_5\left(\sqrt{x^2+5+x}\right) < 1$$

$(x^2 + 5 + x)^{1/2} < 5$ and $x^2 + x + 5 > 0$

$\Rightarrow x^2 + 5 + x < 25$

$\Rightarrow x^2 + x - 20 < 0$

$\Rightarrow (x + 5)(x - 4) < 0$

$\Rightarrow x \in (-5, 4)$

$\therefore n = 8$

Q.26 **(2)**

$$\log_{1/2}\left(x^2 - 6x + 12\right) \geq -2 \qquad \text{.....(i)}$$

For log to be defined, $x^2 - 6x + 12 > 0$

$\Rightarrow (x - 3)^2 + 3 > 0$, which is true $\forall x \in R$.

From (i), $x^2 - 6x + 12 \leq \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow x^2 - 6x + 12 \leq 4 \Rightarrow x^2 - 6x + 8 \leq 0$

$\Rightarrow (x - 2)(x - 4) \leq 0 \Rightarrow 2 \leq x \leq 4 \;\therefore x \in [2, 4]$.

Q.27 (2)

$2x^2 - 4xy + xy - 2y^2 = 7$

$2x(x - 2y) + y(x - 2y) = 7$

$(x - 2y)(2x + y) = 7$

x, y are integers $\Rightarrow x - 2y, 2x + y$ are also integers

Four cases are possible

Case I: $x - 2y = 1, 2x + y = 7 \Rightarrow x = 3, y = 1$

$|x + y| = 4$

Case II : $x - 2y = 7, 2x + y = 1 \Rightarrow x = \frac{9}{5}$ rejected

Case III : $x - 2y = -1, 2x + y = -7$

$\Rightarrow x = -3, y = -1$

$|x + y| = 4$

Case IV : $x - 2y = -7,\ 2x + y = -1 \Rightarrow x = -\frac{9}{5}$ rejected

Hence $|x + y| = 4$

Q.28 (2)

Taking log on both sides,

$(3x^2 - 10x + 3) \log |x - 3| = 0$

$\log |x - 3| = 0$ or $3x^2 - 10x + 3 = 0$

$x - 3 \neq 0$; $|x - 3| = 1$ or $(x - 3)(3x - 1) = 0$

$x \neq 3$; $(x - 3) = \pm 1$ or $x = 3$; $x = \frac{1}{3}$

$\therefore\ x = 2, 4$ or $x =\ [\because\ x \neq 3]$

Hence, three real solutions

Q.29 **(2)**

$[e] - [-\pi]$

here $e \approx 2.7$

$\pi \approx 3.14$

$\therefore\ [2.7] - [-3.14]$

$= 2 - (-4) = 2 + 4 = 6$

Q.30 **(1)**

$$(0.05)^{\log_{\sqrt{20}}(0.1+0.01+........)} = \left(\frac{1}{20}\right)^{2\log_{20}\left(\frac{0.1}{1-0.1}\right)}$$

$$= 20^{-2\log_{20}(1/9)} = 20^{2\log_{20}9} = 20^{\log_{20}9^2} = 9^2 = 81$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

It is a property

Q.2 (2)

x and y are rational and

$(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$

Since real part of both sides are equal and coefficient of irrational parts should be zero in both sides

$\therefore\ x - 2y = 0 \Rightarrow x = 2y$

and $x - y - 1 = 0 \Rightarrow y = 1 \therefore x = 2$

$x = 2,\ y = 1$

Q.3 (1)

Let any two distinct odd number be $(2n + 3)$ and $(2n + 1)$ when $n \in W$

Now According to question $(2n + 3)^2 - (2n + 1)^2$

$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$

$4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8 = 8(n + 1)$

Which is always divisible by 4 & 8.

Q.4 **(1)**

$$4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$$

$$\Rightarrow \left(2^{(x^2+2)}\right)^2 - 9.2^{(x^2+2)} + 8 = 0$$

Put $2^{(x^2+2)^2} = y$. Then $y^2 - 9y + 8 = 0$, which gives $y = 8,\ y = 1$.

when $y = 8 \Rightarrow 2^{x^2+2} = 8 \Rightarrow 2^{x^2+2} = 2^3 \Rightarrow x^2 + 2 = 3$

$\Rightarrow x^2 = 1 \Rightarrow x = 1, -1$.

when $y = 1 \Rightarrow 2^{x^2+2} = 1 \Rightarrow 2^{x^2+2} = 2^\circ$

$\Rightarrow x^2 + 2 = 0 \Rightarrow x^2 = -2$, which is not

Q.5 (D)

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} = ?$$

Let $a - b = A$, $(b - c) = B$, $(c - a) = C$

$[\because A + B + C = 0 \Rightarrow A^3 + B^3 + C^3 = 3ABC]$

$$\therefore\ \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3ABC}{ABC} = 3$$

Q.6 $S = \frac{1}{3}\left[\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + ... + \log\frac{99}{100}\right]$

$$= \frac{1}{3}\left[\log\frac{1}{2}.\frac{2}{3}.\frac{3}{4}..........\frac{99}{100}\right]$$

$$S = \frac{1}{3}\log_{10}\frac{1}{100} = \frac{-2}{3}.\ \text{Ans.}$$

Q.7 (4)

Given, $\log_x \log_{18}(\sqrt{2} + \sqrt{8}) = \frac{1}{3}$

$\Rightarrow \log_{18}(\sqrt{2} + 2\sqrt{2}) = x^{1/3} \Rightarrow \log_{18}3\sqrt{2} = x^{1/3}$

$\Rightarrow (3\sqrt{2})^2 = \left(18^{x^{1/3}}\right)^2 \Rightarrow 18 = 18^{2x^{1/3}}$

$2x^{1/3} = 1 \Rightarrow x^{1/3} = \frac{1}{2} \Rightarrow x = \frac{1}{8} \Rightarrow 1000x = 125$

Q.8 (3)

$\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$

$\Rightarrow x < 0$

$\sqrt{\log_{10}(-x)} = \log_{10}(-x)$

$\Rightarrow \log^2_{10}(-x) - \log_{10}(-x) = 0$

$\Rightarrow \log_{10}(-x) = 0, 1 \Rightarrow x = -1, -10$

two solution

Q.9 (4)

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

Q.10 (2)

$$3^{\log_3(x^2)} - 2x - 3 = 0$$

$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x - 3)(x + 1) = 0$
$\Rightarrow x = -1, 3, (x = -1 \text{ reject } \because x > 0)$
number of values of x is one

Q.11 (3)
$\log_2 7 \Rightarrow \log_2 4 < \log_2 7 < \log_2 8$
$\Rightarrow 2 < \log_2 7 < 3$ i.e. not integer
Let $\log_2 7 = \frac{p}{q}$ (where p and q are coprime)
$\Rightarrow 2^{p/q} = 7 \Rightarrow 2^p = 7^q$

which is not possible so $\log_2 7$ is an irratoinal number

Q.12 **(2)**

$$\log_2 .\log_3 ....\log_{99} \log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}} = \log_2 \log_3 ....\log_{99} 99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}$$

$[\log_{100}100 = 1]$

$$= \log_2 .\log_3 ....\log_{98} 98^{97^{\cdot^{\cdot^{\cdot^{2^1}}}}}$$

$$= \log_2 .\log_3 ....\log_{97} 97^{96^{\cdot^{\cdot^{\cdot^{2^1}}}}} = \log_2 \log_3 3^{2^1}$$

$$= \log_2 2' \log_3 3 = \log_2 2 = 1.$$

Q.13 **(4)**
$\log_2(x + 5) = 6 - x \Rightarrow x + 5 = 2^{6-x} \Rightarrow x + 5 = 64.2^{-x}$
Let $y = x + 5$, $y = 64.2^{-x}$ will intersect at one point.
Number of solutions = 1.

Q.14 (3)
$\Rightarrow \text{antilog}_{16}\, 0.75 = (16)^{0.75}$
$= (16)^{3/4} = (2^4)^{3/4} = 2^3 = 8$

Q.15 (3)
Let $x = 2^m$
$\Rightarrow \log_{10} x = m \log_{10} 2 = 0.3010\, m$
$\therefore$ Char $= [0.3010\, m]$
$\Rightarrow [0.3010\, m] + 1 = 4 \Rightarrow 3 \le 0.3010m < 4$
$\Rightarrow 9 \cdot \ldots\ldots \le m < 13 \cdot \ldots\ldots$
$\therefore m = 10, 11, 12, 13$

Q.16 (3)

$A = \text{Antilog}_{32}(0.8) = (32)^{0.8} = \left(2^5\right)^{\frac{4}{5}} = 2^4 = 16$

$B = 5^{4+1} - 5^4 = 3125 - 625 = 2500$

$C = -\log_7\left(\log_3 \sqrt{\sqrt[7]{9}}\right) = -\log_7\left(\log_3 9^{\frac{1}{14}}\right) =$

1.

So unit digit of A + B + C is 7

Q.17 (1)
Domain $x^2 + 4x - 5 \ge 0 \Rightarrow x \in (-\infty, -5] \cup [1, \infty)$
Case I :
$x \in (-\infty, -5] \cup [1, 3)$
– ve < + ve alsways true
$\therefore x \in (-\infty, -5] \cup [1, 3)$ ... (1)
Case II :
$x \in [3, \infty)$ .. (i)
$x - 3 < \sqrt{x^2 + 4x - 5} \Rightarrow x^2 - 6x + 9 < x^2 + 4x - 5$
$\Rightarrow x > \frac{7}{5}$ ... (ii)
(i) $\cap$ (ii) $x \in [3, \infty)$ ... (2)
(1) $\cup$ (2) $x \in (-\infty, -5] \cup [1, \infty)$

Q.18 (1)
$x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$
$\log_5(x^2 - 4) > 0 \Rightarrow x^2 - 4 > 1 \Rightarrow x^2 - 5 > 0$
$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$
Now
$\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1 \Rightarrow \log_5(x^2 - 4) < 1$
$x^2 - 4 < 5 \Rightarrow x^2 - 9 < 0$, $x \in (-3, 3)$
$\therefore$ **Ans. :** $\left(-3, -\sqrt{5}\right) \cup \left(\sqrt{5}, 3\right)$

Q.19 **(3)**
$\log_{0.04}(x - 1) \ge \log_{0.2}(x - 1)$ ....(i)
For log to be defined $x - 1 > 0 \Rightarrow x > 1$
From (i), $\log_{(0.2)^2}(x-1) \ge \log_{0.2}(x-1)$

$\Rightarrow \frac{1}{2}\log_{0.2}(x-1) \ge \log_{0.2}(x-1) \Rightarrow \sqrt{x-1} \le (x-1)$

$\Rightarrow \sqrt{x-1}(1-\sqrt{x-1}) \le 0 \Rightarrow 1-\sqrt{x-1} \le 0$

$\Rightarrow \sqrt{x-1} \ge 1 \Rightarrow x \ge 2 \therefore x \in [2, \infty)$

Q.20 **(1)**
$\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\log_{0.3}(x-1) < \frac{1}{2}\log_{(0.3)}(x-1)$

$\log_{0.3}(x-1) < \log_{(0.3)}(x-1)^{1/2}$

here base is less than 1, therefore the inequality is reversed
$(x - 1) > (x - 1)^{1/2}$
$(x - 1)^2 > (x - 1)$
$x^2 \; 1 - 2x - x + 1 > 0$
$x^2 - 3x + 2 > 0$
$x^2 - 2x - x + 2 > 0$
$x(x - 2) - 1(x - 2) > 0$
$(x - 1)(x - 2) > 0$
+ – +
1 2
$x \in (-\infty, 1) \cup (2, \infty)$
$\because x - 1 > 0 \Rightarrow x > 1$
than $x \in (2, \infty)$

Q.21 (3)
When $x \le -\frac{3}{4}$ $-4x - 3 - 3x + 4 = 12 - 11 = 7x$

$\Rightarrow\ x = -\dfrac{11}{7}$

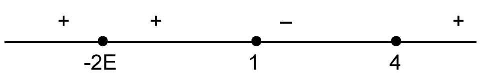
When $-\dfrac{3}{4} < x \le \dfrac{4}{3}$; $4x + 3 - 3x + 4 = 12$
x = 5 will not satisfy

When $x > \dfrac{4}{3}$ $4x + 3 + 3x - 4 = 12$

$7x = 13 \Rightarrow x = \dfrac{13}{7}$

Q.22 (4)
$|x|^2 - 3|x| + 2 = 0 \ \Rightarrow\ (|x| - 2)(|x| - 1) = 0$
$\Rightarrow |x| = 1, 2 \Rightarrow\ x = \pm 1, \pm 2$
$\therefore$ number of real roots is 4.

Q.23 (1)
Since $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x+2)$
$\therefore\ (x^2 - 2x - 8)(x^2 + x - 2) \le 0$
i.e. $(x - 4)(x + 2)(x + 2)(x - 1) \le 0$



$\therefore$ Solution set is $[1,4] \cup \{-2\}$

Q.24 (2)
$||x - 1| - 1| \le 1$
$\Rightarrow -1 \le |x - 1| - 1 \le 1$
$\Rightarrow 0 \le |x - 1| \le 2$
$\Rightarrow -2 \le x - 1 \le 2$
$\Rightarrow -1 \le x \le 3$
$\therefore$ **Ans. :** $x \in [-1, 3]$

Q.25 (4)
$2\{x\}^2 - 5\{x\} + 2 = 0$
$\Rightarrow 2f^2 - 4f - f + 2 = 0$
$\Rightarrow 2f(f - 2) - 1(f - 2) = 0$

$\Rightarrow f = \dfrac{1}{2}, 2,\ f \ne 2\ (0 \le f < 1) \Rightarrow f = \dfrac{1}{2} \Rightarrow \infty$ solution

Q.26 (1)
(i) $-x^2 + 5x - 6 \ge 0 \Rightarrow x \in [2, 3]$

(ii) $2\{x\} < 1 \Rightarrow 0 \le \{x\} < \dfrac{1}{2}$

$\therefore$ by (i) & (ii)

$\therefore\ x \in \left[2, \dfrac{5}{2}\right) \cup \{3\}$

Q.27 (2)
Clearly Domain is $x > 0$ and $x \ne 1$
$x^2 > 0, \quad x \ne 0$

Q.28 (3)

$\sum_{n=1}^{49} f(n) = 0 \Rightarrow \sum_{n=50}^{149} f(n) = 100$

$\Rightarrow f(150) + f(151) = 4 \qquad \Rightarrow \sum_{n=1}^{151} f(n) = 104$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (B)
`$P(4) = k4^3 + 3\cdot4^2 - 3$ & $Q(4) = 2\cdot4^3 - 5\cdot4 + k$
remainder is same
$P(4) = Q(4) \ \Rightarrow\ 64k + 48 - 3 = 128 - 20 + k$

$\Rightarrow\ 63k = 108 - 45 \Rightarrow\ k = \dfrac{63}{63} = 1$

Q.2 (D)

We have, $x = \log_{\frac{3}{5}}\left(\dfrac{5}{3}\right) = -1 \ \Rightarrow\ x = -1$

Let $\log_{0.125} y = \dfrac{-1}{3} \ \Rightarrow\ y = \left(\dfrac{1}{8}\right)^{\frac{-1}{3}} = \left(2^{-3}\right)^{\times\frac{-1}{3}} = 2$
So, y = 2
Hence, $xy = -1(2) = -2$.

Q.3 (D)

$\dfrac{1}{\log_b{}^b + \log_b{}^a + \log_b{}^c} + \dfrac{1}{\log_c{}^c + \log_c{}^a + \log_c{}^b} + \dfrac{1}{\log_a{}^a + \log_a{}^b + \log_a{}^c}$

$= \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc} + \dfrac{1}{\log_a abc}$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

Q.4 (A)

$3^{\log_3 x} + 3^{2\log_3 x} + 3^{3\log_3 x} = 3$
$\Rightarrow\ x + x^2 + x^3 = 3 \ \Rightarrow\ (x - 1) + (x^2 - 1) + (x^3 - 1) = 0$
$\Rightarrow\ (x - 1)(1 + x + 1 + x^2 + x + 1) = 0$
$\Rightarrow\ x = 1\ (\because x^2 + 2x + 3 \ne 0)$

Q.5 (B)
$\log(xy^3) = 1 \qquad \log(x^2y) = 1$
$xy^3 = 10$ ......(1) $\qquad x^2y = 10$ ......(2)
(1) ÷ (2)
$y^2 = x$
$y^5 = 10 \ \Rightarrow\ y = 10^{1/5}$
$x = 10^{2/5}$
$\log(xy)^5 = \log(10^{3/5})^5 = 3$

Q.6 (B)
$\log_3 12 \cdot \log_3 36 - \log_3 4 \cdot \log_3 108 = 2(2\log_3 2 + 1)(\log_3 2 + 1) - (2\log_3 2)(3 + 2\log_3 2)$

Let $\log_3 2 = t$

$\therefore$ $2(2t+1)(t+1) - 2t(2t+3) = 2(2t^2 + 3t + 1 - 2t^2 - 3t) = 2 \times 1 = 2.$

Q.7 (A)

Let $x = \frac{1}{2}\sqrt{150 + \frac{1}{2}\sqrt{150 + \frac{1}{2}\sqrt{150 + \ldots\ldots}}}$

$= \frac{1}{2}\sqrt{150 + x}$

$\Rightarrow 4x^2 - x - 150 = 0 \Rightarrow 4x^2 - 25x + 24x - 150 = 0$

$\Rightarrow (4x - 25)(x + 6) = 0 \Rightarrow x = \frac{25}{4}$

$\because x > 0$

$\therefore$ Given expression $= 7 + \log_{2/5}\left(\frac{25}{4}\right) = 7 - 2 = 5.$

Q.8 (B)

Sol.As, $\sqrt{x} + 1 \ge 1$ (whenever defined)

$\therefore \log_2\left(\sqrt{x} + 1\right) \ge 0 \ \forall x \ge 0$

Also, $1 - \sqrt{x} \le 1$ (whenver defined)

$\therefore \log_3\left(1 - \sqrt{x}\right) \le 0 \ \forall x \in [0, 1)$

So, the given equation will have exactly one real solution i.e., $x = 0$.

Q.9 (D)

$$\begin{cases} \log_8(1) + \log_3(x+2) = \log_3(3-2y) & \ldots\ldots(1) \\ 2^{x+y} - 8^{3-y} = 0 & \ldots\ldots(2) \end{cases}$$

1st gives $\log_3(x + 2) = \log_3(3 - 2y)$

$\therefore$ $x + 2 = 3 - 3y$

$x + 2y = 1$ ......(3)

2nd gives $x + y = (3 - y)3$

$x + y = 9 - 3y$

$x + 4y = 9$ .....(4)

(4) – (3) gives $2y = 8 \Rightarrow y = 4$

$\therefore$ $x = -7$

$\therefore$ $y - x = 11$ but $x = -7$ to be rejected.

Q.10 (D)

$4 \le \log_{1/4}\frac{1}{N} < 5$

$4 \le \log_4 N < 5$

$4^4 \le N < 4^5$

$256 \le N < 1024$

$N = 768$

Q.11 (D)

$\log_{|x|}(x^2 + x + 1) \ge 0$

D : $|x| \ne 0, 1$

case-I : if $|x| < 1$

$x^2 + x + 1 \le 1 \Rightarrow x(x + 1) \le 0$

$x \in (-1, 0)$

case-II : if $|x| > 1$

$x^2 + x + 1 > 1 \Rightarrow x(x + 1) \ge 0$

$\Rightarrow x \in (-\infty, -1) \cup (0, \infty)$

$\therefore x \in (-\infty, -1) \cup (1, \infty)$

Q.12 (A)

Denomender is always positive if $4 - x^3 > 0$

$x^3 - 4 < 0$

$\Rightarrow x < 4^{1/3}$

..... (A)

N^r is positive if $|x + 2| - |x| \ge 0$

$\Rightarrow |x + 2| \ge |x|$ ....

$\Rightarrow x \in [-1, \infty)$

....(B)

$A \cap B \Rightarrow x \in [-1, 4^{1/3})$

Q.13 (A)

$$\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right)} \ge 0$$

$\Rightarrow \sqrt{(x-8)(2-x)} \le 0$

$\left[\because \log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right) < 0\right]$

$\Rightarrow x = 2, 8$ ... (i)

Now $2^{x-3} > 31 \Rightarrow (x - 3) > \log_2 31$

$\Rightarrow x > 3 + \log_2 2^{4.9}$ (approx)

$x > 7.9$

$x = 8$

Q.14 (D)

$\sin x \ne 1$, $2 - \cos^2 x > 1 - \sin x$

$\sin^2 x + \sin x > 0$

$\sin x(\sin x + 1) > 0$

$\Rightarrow \sin x > 0$

$x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Q.15 (A)

$\log_2(x^2 + 1) > 0 \ \forall x \in R - \{0\}$

$\Rightarrow \log_2(4x^2 - x - 1) > \log_2(x^2 + 1)$

$\Rightarrow 4x^2 - x - 1 > x^2 + 1 > 0$

$\Rightarrow 4x^2 - x - 1 > x^2 + 1$

$\Rightarrow 3x^2 - x - 2 > 0$

$\Rightarrow 3x^2 - 3x + 2x - 2 > 0$

$\Rightarrow 3x(x - 1) + 2(x - 1) > 0$

$\Rightarrow x < -\frac{2}{3}$ or $x > 1$

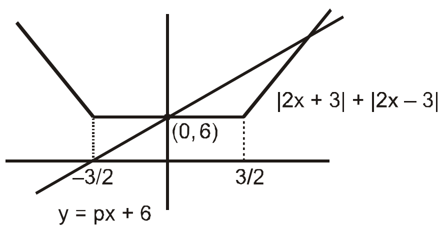
Q.16 (B)

$f(x) = |x-1| + |x-2| + |x-3|$

$$= \begin{cases} -x+1-x+2-x+3 = 6-3x, & x \le 1 \\ x-1-x+2-x+3 = 4-x & 1 < x \le 2 \\ x-1+x-2-x+3 = x & 2 < x \le 3 \\ x-1+x-2+x-3 = 3x-6 & x > 3 \end{cases}$$

min f(x) = 2.

Q.17 (D)



For more than 2 solution p = 0

Q.18 (C)

$[2x] - 3\{2x\} = 1$

$\{2x\} = \dfrac{[2x]-1}{3}$

$0 \le \{2x\} < 1$

$\Rightarrow 1 \le [2x] < 4$

$\Rightarrow [2x] = 1, 2, 3$

if $[2x] = 1$, $\{2x\} = 0$

$\Rightarrow 2x = 1 + 0 \Rightarrow x = \dfrac{1}{2}$

if $[2x] = 2$; $\{2x\} = \dfrac{1}{3}$

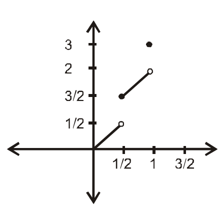
$\Rightarrow 2x = 2 + \dfrac{1}{3} \Rightarrow 2x = \dfrac{7}{3} \Rightarrow x = \dfrac{7}{6}$

if $[2x] = 3$, $\{2x\} = \dfrac{2}{3}$

$\Rightarrow 2x = 3 + \dfrac{2}{3}$ we get three value of x

Q.19 (D)

$[x + [2x]] < 3$

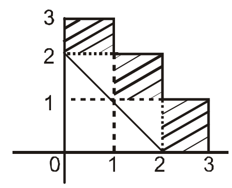


$[x] + [2x] < 3$

True for

$x < 1$

Q.20 (D)



1 + 1 + 1 = 3

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A,B)

$\dfrac{y}{x} = x$

$\Rightarrow y = x^2$ $[\because x \ne 0]$

$\Rightarrow y \ne 0$

$\because x^2 > 0 \Rightarrow y > 0 \quad \therefore y \ne -1$

Q.2 (A, B, C)

$x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$

$\left[\dfrac{3}{4}(\log_2 x)^2 + \log_2 x - \dfrac{5}{4}\right] \log_2 x = \log_2 \sqrt{2} = \dfrac{1}{2}$

Let $\log_2 x = t$

$(3t^2 + 4t - 5)\, t = 2 \Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$t = 1, -2, -\dfrac{1}{3} = \log_2 x$

$x = 2, \dfrac{1}{4}, 2^{-\frac{1}{3}}$

Q.3 (A, B)

$(\log_5 x)^2 + \log_{5x} \dfrac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x} 5 - \log_{5x} x = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1$

$\Rightarrow \dfrac{t^2(1+t) + 1 - t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t(t^2 + t - 2) = 0$

$t(t - 1)(t + 2) = 0$

$t = 0, 1, -2$

$\therefore\ \log_5 x = 0, 1, -2$

$\therefore\ x = 1, 5, \frac{1}{25}$

Q.4 (A,B,C,D)

$= \log_3 135 \log_3 15 - \log_3 5 \log_3 405$

$= \log_3(5 \times 3^3) \cdot \log(5 \times 3) - \log_3 5 . \log_3 (5 \times 3^4)$

$= (\log_3 5 + \log_3 3^3)(\log_3 5 + \log_3 3) - \log_3 5 (\log_3 5 + \log_3 3^4)$ $= (x + 3)(x + 1) - x(x + 4)$ {Let $\log_3 5 = x$}

$= x^2 + 4x + 3 - x^2 - 4x = 3$

which is Prime, rational Integer and natural number

Q.5 (A,B)

$\Rightarrow\ \log_3 xy = 2\log_3 3 + \log_3 2$

$\Rightarrow\ \log_3 xy = \log_3(2 \times 9)\ \Rightarrow\ xy = 18$ .....(i)

and $\log_{27}(x + y) = \frac{2}{3}$ $\Rightarrow\ x + y = 27^{2/3}$

$\Rightarrow\ x + y = 3^2$ $\Rightarrow\ x + y = 9$ ......(ii)

from equation (i) & (ii)

$\therefore\ x^2 - 9x + 18 = 0 \Rightarrow\ (x - 6)(x - 3) = 0$

$\Rightarrow\ x = 6, 3$ so $(x, y) \equiv (6, 3) \equiv (3, 6)$

Q.6 (A,B,C,D)

$\frac{4}{2}\log_x 2 + \frac{\log_x 64}{\log_x 2x}$

$\Rightarrow\ \frac{2x\log_x 2}{x} + \frac{6\log_x 2}{1+\log_x 2} = 3$

Let $\alpha = \log_x 2$

$2\alpha + \frac{6\alpha}{1+\alpha} = 3$

$2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore\ \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \frac{1}{2} \Rightarrow x = 4$ (Integer)

Q.7 (A,B,C,D)

Taking $\log_3$ on both sides

$[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5](\log_3 x) = \frac{3}{2}$

Let $\log_3 x = \alpha$

$\Rightarrow\ \frac{(2\alpha^3 - 9\alpha^2 + 10\alpha)}{2} = \frac{3}{2}$

$\Rightarrow 2\alpha^3 - 9\alpha^2 + 10\alpha - 3 = 0$

$\Rightarrow\ (\alpha - 1)(2\alpha^2 - \alpha + 3) = 0$

$\Rightarrow\ (\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0 \Rightarrow\ \alpha = 1, 3, \frac{1}{2}$

$\therefore\ \log_3 x = 1; \log_3 x = 3 ; \log_3 x = \frac{1}{2}$

$\Rightarrow\ x = 3 ;\ x = 3^3 = 27 ; x = \sqrt{3}$

Exactly three solution, one is irrational solution and every real number is also complex.

Q.8 (A,C)

Let $N_1 = 3^{40}$

$\therefore\ \log_{10} N_1 = 40\log_{10} 3 = 40 \times 0.4771 = 19.084$

So, L = 20

Let $N_2 = 3^{-40}$

$\Rightarrow\ \log_{10} N_2 = -40 \times \log_{10} 3 = -40 \times 0.4771 = -19.084 = -19 - 1 + (-0.084) + 1 = -20 + 0.916$

$= \overline{20}.916$

So, M = 19.

Now, verify alternatives.

Q.9 (A,D)

$x = 100\, e^{23}$ $\Rightarrow \log_{10} x = 2 + 23 \log_{10} e = 2 + 23 \times 0.434 = 11.982$ $\Rightarrow\ N = 12,$

Now $y = e^{-23}$

$\Rightarrow \log_{10} y = -23 \log_{10} e = -9.99$

$\Rightarrow M = 9$

Now verify the options.

Q.10 (C,D)

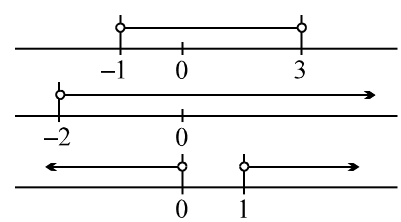
$\log_{x+3}(x^2 - x) < 1$

$x(x - 1) > 0$ $\Rightarrow$ $x > 1$ or $x < 0$ ....(1)

let $x + 3 > 1$ $\Rightarrow$ $x > -2$

here we have $x^2 - x < x + 3$ $\Rightarrow$ $x^2 - 2x - 3 < 0$ $\Rightarrow$ $(x - 3)(x + 1) < 0$

hence $x \in (-1, 0) \cup (1, 3)$ $\Rightarrow$ **(C), (D)**



again,

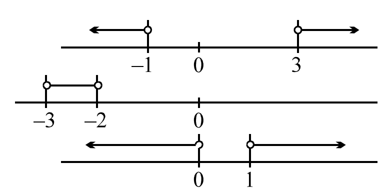
let $0 < x + 3 < 1$

$-3 < x < -2$ ....(1)

then $x^2 - x > x + 3$ $\Rightarrow$ $x^2 - 2x - 3 > 0$

$\Rightarrow$ $(x - 3)(x + 1) > 0$ ....(2)

hence $x \in (-3, -2)$ $\Rightarrow$ **(A)**



Q.11 (A, C)

$\left(\frac{1}{3}\right)^{\{x\}} > \frac{1}{\sqrt{3}}$

$\Rightarrow \{x\} < \frac{1}{2} \Rightarrow \{x\} < 0.5$

$x = \pi \quad \{x\} = 0.14$

$x = -1 + \frac{1}{\sqrt{2}} \Rightarrow [x] = -1 \Rightarrow \{x\} = x - [x] = -1 +$

$\frac{1}{\sqrt{2}} + 1 = \frac{1}{\sqrt{2}} > \frac{1}{2}$

$x = 2 + \frac{1}{9^{1/3}}$

$\Rightarrow \{x\} = \frac{1}{9^{1/3}} < \frac{1}{8^{1/3}} = \frac{1}{2}$

$x = \frac{e}{2} = \frac{2.71}{2} \approx 1.35$

$\Rightarrow \{x\} = 0.35$

Q.12 (A, B, D)

$\frac{1}{2} \le \log_{0.1}x \le 2 \Rightarrow \left(\frac{1}{10}\right)^{1/2} \ge x \ge \left(\frac{1}{10}\right)^{2}$

Q.13 (A, B)

$\log_{100}|x + y| = \frac{1}{2}$

$\therefore\ |x + y| = 100^{1/2} = 10$

$\therefore\ |x + y| = 10$

......(1)

And $\log_{10}y - \log_{10}|x| = \log_{100}4$

$\Rightarrow \log_{10}y - \log_{10}|x| = \log_{(10)^2} 2^2 =$

$\Rightarrow \log_{10}\frac{y}{|x|} = \log_{10}2$

$\therefore\ \frac{y}{|x|} = 2 \Rightarrow y = 2|x|$

.....(2)

From (1) & (2) $|x + 2|x|| = 10$

Case I :

If $x > 0$

$3x = 10$

$\Rightarrow x = \frac{10}{3}$ & $y = \frac{20}{3}$

Case II

If $x < 0$

$\therefore\ x = -10$ & $y = 20$

Q.14 (B, C)

$|x - 6| + |x + 6| = 12$

Case-I : $x < -6$

$-x + 6 - x - 6 = 12$

$\Rightarrow x = -6$ (rejected)

Case-II : $-6 \le x \le 6$

$-x + 6 + x + 6 = 12$

$12 = 12$

$\therefore x \in [-6, 6]$

Case-III : $x > 6$

$x - 6 + x + 6 = 12$

$\Rightarrow x = 6$ (rejected)

$\therefore x \in [-6, 6]$

$\therefore$ Number of integral solutions is 13,

$\{-6, -5, \ldots\ldots 5, 6\}$

and sum of these integral solutions is zero.

Q.15 (A, B)

$\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2$

$\Rightarrow |2\log_3 x - 2| - |\log_3 x - 2| = 2$

case I If $\log_3 x - 2 \ge 0 \quad \Rightarrow \log_3 x \ge 2$

Then $2\log_3 x - 2 - \log_3 x + 2 = 2$

$\Rightarrow \log_3 x = 2$

$\therefore\ \log_3 x = 2 \Rightarrow x = 3^2 = 9$

$\Rightarrow x = 9$

case II $1 \le \log_3 x < 2$

$\therefore\ 2\log_3 x - 2 + \log_3 x - 2 = 2$

$\Rightarrow 3\log_3 x = 6 \Rightarrow \log_3 x = 2$

which is not possible

case III If $\log_3 x < 1$

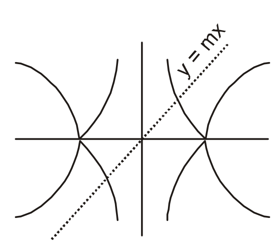
the $-2\log_3 x + 2 + \log_3 x - 2 = 2$

$-\log_3 x = 2$

$\Rightarrow \log_3 x = -2$

$\therefore\ x = 3^{-2} = \frac{1}{9} \Rightarrow x = 9, \frac{1}{9}$

Q.16 (A, B, C)



for m = 1, –1, 2

we will get two solution

Q.17 (A, B, C, D)

$[2 - x] + 2[x - 1] \ge 0$

$[1 - x] + 2[x - 1] + 1 \ge 0$

case-I : if $x \notin I$

$-[x-1]-1+2[x-1]+1 \geq 0$
$\Rightarrow [x-1] \geq 0$
$\Rightarrow x \geq 1$
case-II : if $x \in I$
$-[x-1]+2[x-1]+1 \geq 0$
$\Rightarrow (x-1) \geq -1$
$x = 0, 1, 2 \ldots\ldots\ldots\infty$
$\therefore x \in [1, \infty) \cup \{0\}$

Q.18 (A, C)
$f(x) = \cos 9x + \cos(-10x)$
$f(x) = \cos 9x + \cos 10x$

$f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = -1$

$f(\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$
$f(-\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

$f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{10\pi}{4} = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

Comprehension # 1 (Q. No. 19 to 21)

Q.19 (B)
Q.20 (A)
Q.21 (A)
Sol. **(19, 20, 21)**

Case I when $1+\frac{3}{x} \geq 0 \quad \Rightarrow \frac{x+3}{x} \geq 0$

$\Rightarrow x \in (-\infty, -3] \cup (0, \infty)$ ..... (i)
Now for above interval

$\left|1+\frac{3}{x}\right| > 2 \Rightarrow 1 + \frac{3}{x} > 2 \quad \Rightarrow 1 - \frac{3}{x} < 0$

$\Rightarrow x \in (0, 3)$
..... (ii)
from (i) and (ii) $x \in (0, 3)$ ..... (A)

Case II when $1+\frac{3}{x} < 0$ then $x \in (-3, 0)$

..... (iii)

Now $\left|1+\frac{3}{x}\right| > 2 \Rightarrow -\left(1+\frac{3}{x}\right) > 2$

$\Rightarrow 3+\frac{3}{x} < 0$

$\Rightarrow x \in (-1, 0)$
..... (iv)
from (iii) and (iv) $x \in (-1, 0)$.... (B)
$\therefore$ Solution set for

$\left|1+\frac{3}{x}\right| > 2$ is

$x \in (-1, 0) \cup (0, 3)$
by comparing
it with $(a, 0) \cup (0, b)$ we get $a = -1$, $b = 3$
$\therefore |a+b| = |-1+3| = 2$
$\because |a+b| = 2$

$\therefore 5^{\log_3|a+b|} + \log_2|a+b| - 2^{\log_3 5}$

$\Rightarrow 5^{\log_3 2} + \log_2 2 - 2^{\log_3 5} = 1$

$(x+1)^2 < (7x-3)$
$\Rightarrow x^2 + 2x + 1 < (7x-3)$
$\Rightarrow x^2 - 5x + 4 < 0$
$\Rightarrow (x-1)(x-4) < 0$
$\Rightarrow x \in (1, 4)$
$\therefore c = 1, d = 4$
$\therefore a + b + c + d = -1 + 3 + 1 + 4 = 7$

Comprehension # 2 (Q. no. 22 to 24)

Q.22 (C)
$\log(3^{1/2x} . 3) = \log(108 - 3^{1/x})$

$3^{1+\frac{1}{2x}} = 108 - 3^{1/x}$

Let $3^{1/2x} = t$
$3t = 108 - t^2$
On solving, we get

$t = 9 \Rightarrow 3^{1/2x} = 9 \Rightarrow \frac{1}{2x} = 2$

$\therefore x = 1/4 \Rightarrow A = 1/4$

Q.23 (A)
$5^{\log x - \log^2 x} = 5^{-3} . 5^{\log x - 1}$

$\log x - \log^2 x = -3 + \log x - 1$
$\log^2 x = 4$
$\log x = \pm 2$

$x = 100, \frac{1}{100}$

$B = 2$

Q.24 (C)
$\because 10^{(\ell n x)^2 + 6\ell n x - 16} = 10^0$

Let $t = \ln x$
$t^2 + 6t - 16 = 0 \Rightarrow (t+8)(t-2) = 0$
$t = -8, t = 2; \ell n\, x - 2$
$x = e^2 = C$ [$\because$ C > 1; $\ell n\, x = -8$ is rejected)

$\therefore d = \log \frac{1}{2} e^{-2\ln 4} = 4$

Total Distance = 8

Comprehension # 03 (Q. no. 25 to 27)

Q.25 (B)
Q.26 (A)

Q.27 (D)

Sol. $x = 3^{\log 5 - \log 7}$

$y = 5^{\log 7 - \log 3}$

$z = 7^{\log 3 - \log 5}$

$\therefore\ x \cdot y \cdot z = 1$

$\therefore$ **A = 1**

$\log_2 (6 \log_2 |x| - 3) - \log_2 (4 \log |x| - 5) = \log_2 3$

$$\frac{6\log_2|x|-3}{4\log_2|x|-5} = 3$$

let $\log_2 |x| = 1$

$$\therefore\ \frac{6t-3}{4t-5} = 3$$

$6t - 3 = 12t - 15,\ \ 6t = 12$

$\therefore\ t = 2,\ \ \log_2 |x| = 2,\qquad |x| = 4$

$\therefore\ x = \pm 4$

B = 16 + 16 = 32

$\log_2 (\log_2 3) + \log_2 (\log_3 4) + \log_2 (\log_4 5) + \log_2 (\log_5 6) + \log_2 (\log_6 7) + \log_2 (\log_7 8)$

$= \log_2 (\log_2 8) = \log_2 3$

$\therefore$ **C = 1**

Q.28 (A) → (r), (B) → (p), (C) → (q), (D) → (s)

Sol. (A) $\log_4 (x + 1)(x + 8) = \dfrac{3}{2} \Rightarrow x^2 + 9x + 8 = 8$

$\Rightarrow x = 0, -9$ but $x = -9$ is extraneous.

(B) $x < 0 \Rightarrow 5 - 2x = 6 \qquad \Rightarrow x = -\dfrac{1}{2}$

(C) $3 \log_2 \dfrac{3^4}{2^4.5} + 5 \log_2 \dfrac{5^2}{2^3.3} + 7 \log_2 \dfrac{2^4}{3.5}$

$= 3(4 \log_2 3 - 4 \log_2 2 - \log_2 5) + 5(2 \log_2 5 - 3 \log_2 2 - \log_2 3) + 7 (4 \log_2 2 - \log_2 3 - \log_2 5)$

$= \log_2 2 = 1$

(D) $2\left(-\dfrac{1}{2}\right)^5 - \left(-\dfrac{1}{2}\right)^3 + \left(-\dfrac{1}{2}\right)^2 + 1$

$= -\dfrac{1}{16} + \dfrac{1}{8} + \dfrac{1}{4} + 1$

$= \dfrac{-1+2+4+16}{16} = \dfrac{21}{16}$

Q.29 (A) → (p), (t) → (p), (C) → (s), (D) → (q)

(A) $\sqrt[3]{5^{\log_5 7} + \dfrac{1}{\sqrt{-\log_{10} 1/10}}} = \sqrt[3]{7 + \dfrac{1}{\sqrt{1}}} =$

$\sqrt[3]{8} = 2$ **Ans.**

(B) As $\log_b 3 = 4 \Rightarrow 3 = b^4$ and $\log_{b^2} 27 = \dfrac{3a}{2} \Rightarrow$

$\log_{\sqrt{3}} 27 = \dfrac{3a}{2} \Rightarrow a = 4.$

Hence, $(a^2 - b^4) = 16 - 3 = 13.$

(C) Clearly, $d = 12,\ c = 6$

So, $(d - c) = 12 - 6 = 6.$

(D) $N = \text{antilog}_3\left(\log_6\left(\sqrt{5}\right)^{\log_5 1296}\right) = \text{antilog}_3\left(\log_6 (5)^{\log_5 36}\right) = \text{antilog}_3\left(\log_6 36\right) = \text{antilog}_3(2)$

$N = 9$

characteristic of $\log_2 9 = 3.$

Q.30 (A) → (q), (B) → (t), (C) → (s), (D) → (p)

(A) We have $y = \log_a x,\ x \in [a, 2a]$

Maximum value of $\log_a x$ occurs when $x = 2a$ and minimum value of $\log_a x$ occurs when $x = a$

$\therefore\ p_a = \log_a 2 + 1$ and $q_a = 1$

Given $p_a - q_a = \dfrac{1}{2} \Rightarrow \log_a 2 + 1 - 1 = \dfrac{1}{2} \Rightarrow \log_a 2 = \dfrac{1}{2} \Rightarrow 2 = \sqrt{a}$

Hence $a = 4$. **Ans.**

(B) We have $\displaystyle\sum_{n=1}^{1023} \log_2\left(1+\frac{1}{n}\right)$

$= \log_2\left(1+\dfrac{1}{1}\right) + \log_2\left(1+\dfrac{1}{2}\right) + \ldots\ldots + \log_2\left(1+\dfrac{1}{1023}\right)$

$= \log_2\left(\dfrac{2}{1}\times\dfrac{3}{2}\times\dfrac{4}{3}\times\ldots\ldots\times\dfrac{1024}{1023}\right)$

$= \log_2 1024 = 10$ **Ans.**

(C) A

$= \log_{\sqrt{3}}(3)^{1/2} + \log_{\sqrt{3}}(3)^{1/4} + \log_{\sqrt{3}}(3)^{1/8} + \log_{\sqrt{3}}(3)^{1/16}$

$= 1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8} = \dfrac{8+4+2+1}{8} = \dfrac{15}{8}$

$\therefore \quad \log_{\sqrt{2}}(8A+1) = \log_{\sqrt{2}}(16) = 8$

Ans.]

(D)

$$\left(\log_2(x-1)+\log_2 x\right)^2 - 4\log_2(x-1)\log_2 x = 1$$

$\Rightarrow \quad \left(\log_2(x-1)-\log_2 x\right)^2 = 1$

$\Rightarrow \quad \log_2(x-1) - \log_2 x = \pm 1$

(i) If $\log_2(x-1) - \log_2 x = 1$

$\Rightarrow \dfrac{x-1}{x} = 2 \Rightarrow x - 1 = 2x$

$\therefore \quad x = -1$ (rejected)

$\therefore$ no value of x .

(ii) If $\log_2(x-1) - \log_2 x = -1$

$\Rightarrow \dfrac{x-1}{x} = \dfrac{1}{2} \Rightarrow 2x - 2 = x \quad \Rightarrow$

x = 2 is the only value satisfy the given equation.

Hence **Q, R, S, T**]

NUMERICAL VALUE BASED

Q.1 [3]

$5^x . \sqrt[x]{8^{x-1}} = 500 \Rightarrow 5^x . 8^{\frac{x-1}{x}} = 5^3 . 2^2$

$\Rightarrow 5^{x-3}\, 2^{\frac{3x-3}{x}-2} = 1 \Rightarrow 5^{(x-3)}\, 2^{\frac{3x-3}{x}-2} = 1$

5 & 2 are coprime no. If their multiply is one. So individual power of 5 & 2 is zero

$x - 3 = 0, \ \dfrac{x-3}{x} = 0 \Rightarrow x = 3$

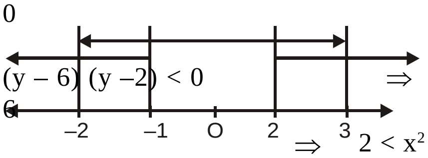
Q.2 [8]

$(x^2 - x - 1)(x^2 - x - 7) + 5 < 0$

Let $x^2 - x = y$

$(y-1)(y-7) + 5 < 0 \quad \Rightarrow \quad y^2 - 8y + 12 < 0$

$(y-6)(y-2) < 0 \quad \Rightarrow \quad 2 < y < 6$



$\Rightarrow \quad 2 < x^2 - x < 6$

$x^2 - x - 2 > 0$ and $x^2 - x - 6 < 0$

$(x-2)(x+1) > 0$ and $(x-3)(x+2) < 0$

$x \in (-2,-1) \cup (2, \ 3)$

Q.3 **[0]**

$x^2 - 16 \geq 0$

$\therefore \quad (x-4)(x+4) \geq 0$

$\therefore \quad x \in (-\infty, -4] \cup [4, \infty)$ ....(1)

Now $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x-3)(x+3)} \leq$

$x \in (-3, 3)$

...........(2)

By (1) and (2) $\quad x \in \{-4, 4\}$

Q.4 [6]

For domain $\quad 21 - 4x - x^2 \geq 0$

$\Rightarrow \quad x^2 + 4x - 21 \leq 0$

$\Rightarrow \quad (x+7)(x-3) \leq 0 \quad \Rightarrow \quad x \in [-7, 3]$

case-I : $-7 \leq x < -1$ then $1 - \sqrt{21 - 4x - x^2} \leq 0$

$\Rightarrow 1 \leq \sqrt{21 - 4x - x^2}$

$\Rightarrow \quad x^2 + 4x - 20 \leq 0 \Rightarrow (x+2)^2 - 24 \leq 0$

$\Rightarrow (x + 2 + 2\sqrt{6})(x + 2 - 2\sqrt{6}) \leq 0$

$\Rightarrow x \in [-2 - 2\sqrt{6}, 2\sqrt{6} - 2]$

$\therefore \quad x \in [-2 - 2\sqrt{6}, -1)$

case-II : $-1 < x \leq 3$ then $1 \geq 21 - 4x - x^2$

$\Rightarrow \quad x^2 + 4x - 20 \geq 0 \Rightarrow$

$x \in (-\infty, -2 - 2\sqrt{6}] \cup [2\sqrt{6} - 2, \infty)$

$\therefore x \in \left[2\sqrt{6} - 2, 3\right]$

$x \in \left[-2 - 2\sqrt{6}, -1\right) \cup \left[2\sqrt{6} - 2, 3\right]$

hence the integers satisfying this inequality are –6, –5, –4, –3, –2, 3 i.e. they are 6 in number.

Q.5 [24]

$\alpha = \log_{10} 15 = 1 + \log_{10} 3 - \log_{10} 2$

$\beta = 4 \log_{10} 2$

$4 \log_{10} 3 = 4\alpha + \beta - 4$

we required those elements of A which can be written in terms of $\log_{10} 2$ & $\log_{10} 3$.

i.e. number of integers which are divisible 2 or 3, or 5 only.

required numbers = 24

Q.6 [0]

$[x]^2 + (x)^2 < 4$

if $\quad x \in I$, then $[x] = (x) = x$

$\therefore \quad x^2 < 2$ i.e. $-\sqrt{2} < x < \sqrt{2}$ i.e.

$x = -1, 0, 1$

if $\quad x \notin I$, then $(x) = 1 + [x]$

$\therefore \quad [x]^2 + (1 + [x])^2 < 4$

i.e. $\quad 2[x]^2 + 2[x] - 3 < 0$

i.e. $\frac{-2-\sqrt{28}}{4} < [x] < \frac{-2+\sqrt{28}}{4}$

i.e. $\frac{-1-\sqrt{7}}{2} < [x] < \frac{-1+\sqrt{7}}{2}$

i.e. $[x] = -1, 0$

$\therefore$ Solution set is $[-1, 1]$

$\therefore$ $\lambda + \mu = 0$

Q.7 **[2]**

a, b $\in$ R greater than one

$\exists\ c \in R^+ \ \&\ c \neq 1$

such that $2(\log_a c + \log_b c) = 9 \log_{ab} c$

$\Rightarrow 2\left(\log_a c + \frac{\log_a c}{\log_a b}\right) = \frac{9\log_a c}{(1+\log_a b)}$

$\Rightarrow 2\log_a c\left(\frac{1+\log_a b}{\log_a b}\right) = \frac{9\log_a c}{(1+\log_a b)}$

$\Rightarrow 2(1+\log_a b)^2 = 9\log_a b \quad \{A = \log_a b\}$

$\Rightarrow 2A^2 + 4A + 2 = 9A \Rightarrow 2A^2 - 5A + 2 = 0$

$\Rightarrow 2A^2 - 4A - A + 2 = 0$

$\Rightarrow 2A(A-2) - 1(A-2) = 0 \Rightarrow A = 2$ or $A = \frac{1}{2}$

Largest value of $A = \log_a b$ is 2.

Q.8 **[9]**

$xy + 2z = \left(\log_2 3 - 4\log_{\left(ln\frac{5}{4}\right)} 3\right)\ 2\log_3\left(ln\frac{5}{4}\right)$

$+\ 2\log_2\left(\log_{\frac{5}{4}} e\right)$

$= 2\log_2\left(ln\frac{5}{4}\right) - 8 - 2\log_2\left(ln\frac{5}{4}\right) = -8$

$\therefore$ $|xy + 2z| = 8$]

Q.9 **[7]**

$2^{ln\,x} = 3^{ln\,y}$ .....(1)

Taking ln both the sides

$ln\,x \cdot ln\,2 = ln\,y \cdot ln\,3 \Rightarrow$

$ln\,x = \log_2 3 \cdot ln\,y$

Put in eqaution

$ln^2 x = ln^3 y \Rightarrow ln\,y = \log_2^2 3$

$\Rightarrow ln\,x = \log_2^3 3 \Rightarrow \log_2 3 + \log_2 3 = \log_2 9$

$\Rightarrow a = 9, b = 2$

$|a - b| = |9 - 2| = 7$

Q.10 **[9]**

Given equation is written as

$|2\log_3 x - 2| - |\log_3 x - 2| = 2$

Let $\log_3 x = t$

$\therefore 2|t-1| - |t-2| = 2$

Case-I: If $t \geq 2 \Rightarrow 2(t-1) - (t-2) = 2 \Rightarrow t = 2 \Rightarrow x = 9$

Case-II: If $1 < t < 2 \Rightarrow 2(t-1) + (t-2) = 2 \Rightarrow t = 2 \Rightarrow x = 9$

Case-III: If $t < 1 \Rightarrow -2(t-1) + t - 2 = 2 \Rightarrow t = -2 \Rightarrow x = \frac{1}{9}$.]

KVPY PREVIOUS YEAR'S

Q.1 (A)
Q.2 (C)
Q.3 (A)
Q.4 (B)
Q.5 (B)
Q.6 (B)
Q.7 (C)
Q.8 (B)
Q.9 (A)
Q.10 (A)
Q.11 (C)
Q.12 (D)
Q.13 (C)
Q.14 (A)
Q.15 (D)
Q.16 (C)
Q.17 (C)
Q.18 (C)
Q.19 (B)
Q.20 (A)
Q.21 (D)
Q.22 (D)
Q23 (B)
Q.24 (C)
Q.25 (B)
Q.26 (C)
Q.27 (B)
Q.28 (C)
Q.29 (B)
Q.30 (A)
Q.31 (D)
Q.32 (B)
Q.32 (C)
Q.34 (A)
Q.35 (C)
Q.36 (D)
Q.37 (D)
Q.38 (C)
Q.39 (C)

Q.40 (D)
Q.41 (D)
Q.42 (D)
Q.43 (B)
Q.44 (A)
Q.45 (B)
Q.46 (C)
Q.47 **(B)**
Q.48 (C)
Q.49 (D)
Q.50 **(A)**
Q.51 **(B)**
Q.52 (D)
Q.53 (B)
Q.54 **(D)**

$165 = 3 \times 5 \times 11$

$\because$ x + y divides xn + yn

$\therefore$ 32 + 1 divides 3211 + 111

Hence N_1 is multiple of 33, simultaneously unit digit in N1 is 9 so it not the multiple of 5

Hence HCF of N_1 & N_2 is 33

Q.55 **(B)**

Obviously $p_4 = 2$ & one of p_1, p_2 is also 2 (say p_2)

so $p = p_1 + 2 = p_3 - 2$

$\Rightarrow$ p_1, p, p_3 are 3 consecutive odd numbers

$\Rightarrow$ atleast one of them is divisible by 3

$\Rightarrow$ $p_1 = 3 \Rightarrow p_2 = 5$ and $p_3 = 7$

Hence only one special prime exists

Q.56 **(C)**

$\sqrt{p}+\sqrt{q}+\sqrt{r} \in Q$, p,q,r t $\in$ Q

let $\sqrt{p}+\sqrt{q}+\sqrt{r} = t$

$\sqrt{p}+\sqrt{q} = t - \sqrt{r}$

$p + q + 2\sqrt{pq} = t^2 + r - 2t\sqrt{r}$

$\sqrt{pq} + t\sqrt{r} \in Q = \lambda\ \lambda \in Q$

$\sqrt{pq} = \lambda - t\sqrt{r}$

$pq = \lambda^2 + t^2 r - 2\lambda t\sqrt{r}$

$\Rightarrow \sqrt{r} \in Q$ similarly $\sqrt{p}$ and $\sqrt{q} \in Q$

hence $\sqrt{p}, \sqrt{q}, \sqrt{r} \in Q$

Q.57 **(B)**

x3 + y3 = 65

(x + y) (x2 + y2 – xy) = 65 × 1

= 13 × 5

= 5 × 13

= 1 × 65

clearly $x^2 + y^2 - xy > 0$

C-1 : $x + y = 5$ and $x^2 + y^2 - xy = 13$

$x^2 + (5 - x)^2 - x(5 - x) = 13$

$3x^2 - 15x + 12 = 0$

$x^2 - 5x + 4 = 0 \Rightarrow x = 1,4$

(x,y) = (1,4) and (4,1)

C-2 : $x + y = 13$ and $x^2 + y^2 - xy = 5$

$x^2 + (13 - x)2 - x(13 - x) = 5$

$3x^2 . 39x + 164 = 0$, $x \notin I$ (Not possible)

C-3 : $x + y = 1$ and $x^2 + y^2 - xy = 65$

$x^2 + (1 - x)^2 . x(1 - x) = 65$

$3x^2 - 3x - 64 = 0$, $x \notin I$ (Not possible)

C-4 : $x + y = 65$ and $x^2 + y^2 . xy = 65$

No solution

so two ordered pair satisfy the relation

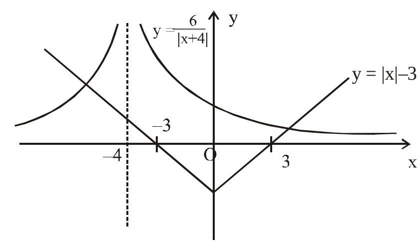
JEE-MAIN
PREVIOUS YEAR'S

Q.1 **(2)**

$x \neq -4$

$(|x| - 3)(|x + 4|) = 6$

$\Rightarrow |x| - 3 = \frac{6}{|x+4|}$



No. of solutions = 2

Q.2 **(1)**

$$y = 4 + \frac{1}{\left(5 + \frac{1}{y}\right)}$$

$$y - 4 = \frac{y}{(5y+1)}$$

$5y^2 - 20y - 4 = 0$

$$y = \frac{20 + \sqrt{480}}{10}$$

$$y = \frac{20 - \sqrt{480}}{10} \to \text{rejected}$$

$$y = 2 + \sqrt{\frac{480}{100}}$$

Correct with Option (A)

Q.3 (1)

Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + ...\infty}}}}$

So, $x = 3 + \dfrac{1}{4 + \dfrac{1}{x}} = 3 + \dfrac{1}{\dfrac{4x+1}{x}}$

$\Rightarrow (x-3) = \dfrac{x}{(4x+1)}$

$\Rightarrow (4x + 1)(x – 3) = x$

$\Rightarrow 4x2 – 12x + x – 3 = x$

$\Rightarrow 4x2 – 12x – 3 = 0$

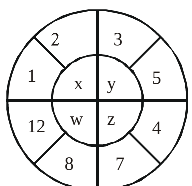
$x = \dfrac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \dfrac{12 \pm \sqrt{12(16)}}{8}$

$= \dfrac{12 \pm 4 \times 2\sqrt{3}}{8} = \dfrac{3 \pm 2\sqrt{3}}{2}$

$x = \dfrac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$.

So, $x = 1.5 + \sqrt{3}$

Q.4 (4)



$x = (2 - 1)1! = 1$

$w = (12 - 8)4! = 424$

$z = (7 - 4)3! = 36$

hence $y = (5 - 3)2! = 22$

Q.5 **[0]**

$P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now P(x) is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x2 + x + 1)$

$P(w) = 0 = P(w2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w – w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

Q.6 [13]

Q.7 [924]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 [**4**]

Let $\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}......}} = t \Rightarrow \sqrt{4 - \dfrac{1}{3\sqrt{2}}t}$

$= t \Rightarrow 4 - \dfrac{1}{3\sqrt{2}} t = t^2 \Rightarrow$

$t^2 + \dfrac{1}{3\sqrt{2}} t - 4 = 0 \Rightarrow 3\sqrt{2}\, t^2 + t - 12\sqrt{2} = 0 \Rightarrow t =$

$\dfrac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \dfrac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \dfrac{16}{6\sqrt{2}}, \dfrac{-18}{6\sqrt{2}} \Rightarrow t = \dfrac{8}{3\sqrt{2}}, \dfrac{-3}{\sqrt{2}}$ and $\dfrac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\dfrac{1}{3\sqrt{2}} \times \dfrac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\dfrac{4}{9}\right) = 6 +$

$\log_{3/2}\left(\left(\dfrac{2}{3}\right)^2\right) = 6 - 2 = 4$

Q.2 **(A, B, C)**

$3^x = 4^{x-1} \Rightarrow x = (x - 1)\log_3 4 \Rightarrow x(1 - 2\log_3 2)$

$= -2\log_3 2$

$x = \dfrac{2\log_3 2}{2\log_3 2 - 1}$ **Ans. (A)**

Again $x \log_2 3 = (x - 1) \cdot 2 \Rightarrow x(\log_2 3 - 2) = -2$

$\Rightarrow x = \dfrac{2}{2 - \log_2 3}$ **Ans. (B)**

$x = \dfrac{1}{1 - \frac{1}{2}\log_2 3} = \dfrac{1}{1 - \log_4 3}$ **Ans.(C)**

Q.3 [8]

$\log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$

$= (\log_2 9)^{2\log^2_{\log 2^9}} \times 7^{\frac{1}{2}\log_7 4}$

$= 4 \times 2 = 8$

Trigonimetric Ration and Identities

EXERCISES

ELEMENTARY

Q.1 (4)

We have,

$\sin\theta + \text{cosec}\,\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

Required value of $\sin^{10}\theta + \text{cosec}^{10}\theta = (1)^{10} + \dfrac{1}{(1)^{10}} = 2$.

Q.2 (4)

$\tan A + \cot A = 4$

$\Rightarrow \tan^2 A + \cot^2 A + 2\tan A\cot A = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14 \Rightarrow \tan^4 A + \cot^4 A + 2 = 196$

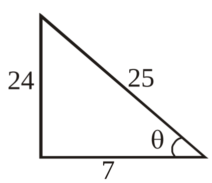
$\Rightarrow \tan^4 A + \cot^4 A = 194.$

Q.3 (2)

The true relation is $\sin 1 > \sin 1^\circ$

Since value of $\sin\theta$ is increasing $\left[0 \to \dfrac{\pi}{2}\right]$.

Q.4 (3)



$\text{Sec}\theta = \dfrac{25}{7}$

$\tan\theta = \dfrac{24}{7}$

$\text{Sec}\theta + \tan\theta = \dfrac{25}{7} + \dfrac{24}{7} = \dfrac{25+24}{7} = \dfrac{49}{7}$

Lies in second Quadrent than $\sec\theta + \tan\theta = -7$

Q.5 (2)

$\cos A + \sin(270^\circ + A) - \sin(270^\circ - A) + \cos(180^\circ + A)$

$\cos A - \cos A + \cos A - \cos A = 0$

Q.6 (1)

$\text{Sin}\,(\pi + \theta).\ \sin(\pi - \theta).\ \text{cosec}^2\theta$

$= (-\sin\theta)(\sin\theta)\,\text{cosec}^2\theta$

$= -\sin^2\theta\,\text{cosec}^2\theta = -1$

Q.7 (1)

$\cos(540^\circ - \theta) - \sin(630^\circ - \theta)$

$= -\cos\theta - (-\cos\theta) = 0$

Q.8 (4)

$e^{\log_{10}\tan 1^\circ + \log_{10}\tan 2^\circ + \log_{10}\tan 3^\circ + \ldots\ldots\ldots + \log_{10}\tan 89^\circ}$

$= e^{\log_{10}(\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ)} = e^{\log_{10} 1} = e^\circ = 1$

Q.9 (4)

We know that $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$= \dfrac{1}{\sqrt{10}}\sqrt{1-\dfrac{1}{5}} + \dfrac{1}{\sqrt{5}}\sqrt{1-\dfrac{1}{10}}$

$= \dfrac{1}{\sqrt{10}}\sqrt{\dfrac{4}{5}} + \dfrac{1}{\sqrt{5}}\sqrt{\dfrac{9}{10}} = \dfrac{1}{\sqrt{50}}(2+3) = \dfrac{5}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$

$\sin(A+B) = \sin\dfrac{\pi}{4}$

Hence, $A + B = \dfrac{\pi}{4}$.

Q.10 (2)

We have $\tan A = -\dfrac{1}{2}$ and $\tan B = -\dfrac{1}{3}$

Now, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B} = \dfrac{-\dfrac{1}{2} - \dfrac{1}{3}}{1 - \dfrac{1}{2}.\dfrac{1}{3}} = -1$

$\Rightarrow \tan(A+B) = \tan\dfrac{3\pi}{4}$. Hence, $A + B = \dfrac{3\pi}{4}$.

Q.11 (3)

$2\tan(A-B) = 2\left(\dfrac{\tan A - \tan B}{1 + \tan A\tan B}\right)$

$= 2\dfrac{(2\tan B + \cot B - \tan B)}{1 + (2\tan B + \cot B)\tan B} = 2\dfrac{\tan B + \cot B}{2(1+\tan^2 B)}$

$= \dfrac{\cot B(\tan^2 B + 1)}{(1 + \tan^2 B)} = \cot B$

Q.12 (2)

$\dfrac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} = \dfrac{2\sin(A+B)\sin(A-B)}{\sin 2A - \sin 2B}$

$= \dfrac{2\sin(A+B)\sin(A-B)}{2\cos(A+B)\sin(A-B)} = \tan(A+B)$.

Q.13 (1)

Since $\tan 3A = \dfrac{\tan A + \tan 2A}{1 - \tan A\ \tan 2A}$

$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A\tan 2A\ \tan A$

Q.14 (2)

We have $\cos\theta = \frac{3}{5}$ and $\cos\phi = \frac{4}{5}$.

Therefore $\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$= \frac{3}{5}.\frac{4}{5} + \frac{4}{5}.\frac{3}{5} = \frac{24}{25}$

But $2\cos^2\left(\frac{\theta-\phi}{2}\right) = 1+\cos(\theta-\phi) = 1+\frac{24}{25} = \frac{49}{50}$

$\therefore \cos^2\left(\frac{\theta-\phi}{2}\right) = \frac{49}{50}$. Hence, $\cos\left(\frac{\theta-\phi}{2}\right) = \frac{7}{5\sqrt{2}}$.

Q.15 (1)

$\sin 4\theta = 2\sin 2\theta\cos 2\theta$

$= 2.2\sin\theta\cos\theta(1-2\sin^2\theta)$

$= 4\sin\theta(1-2\sin^2\theta)\sqrt{1-\sin^2\theta}$

Q.16 (3)

$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2.2\cos^2 2\theta}}$

$= \sqrt{2+2\cos 2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta \cdot$

Q.17 (3)

$\frac{1-\tan^2 15^o}{1+\tan^2 15^o} = \cos(2\times 15^o) = \cos 30^o$

Q.18 (4)

$\tan^2\theta = 2\tan^2\phi + 1$ ... (i)

$\cos 2\theta + \sin^2\phi = \frac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\phi$

$= \frac{1-2\tan^2\phi-1}{1+2\tan^2\phi+1} + \sin^2\phi = \frac{-2\tan^2\phi}{2(1+\tan^2\phi)} + \sin^2\phi$

$= -\sin^2\phi + \sin^2\phi = 0$

which is independent of ϕ

Q.19 (2)

We have $x + \frac{1}{x} = 2\cos\theta$,

Now $x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= (2\cos\theta)^3 - 3(2\cos\theta) = 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$.

Trick : Put $x = 1 \Rightarrow \theta = 0^\circ$

Then $x^3 + \frac{1}{x^3} = 2 = 2\cos 3\theta$

Q.20 (2)

We have $\tan A = \frac{1}{2}$

$\Rightarrow \tan 3A = \frac{3\tan A - \tan^3 A}{1-\tan^2 A} = \frac{3.\frac{1}{2}-\frac{1}{8}}{1-3.\frac{1}{4}} = \frac{12-1}{2} = \frac{11}{2}$

Q.21 (3)

$\tan x + \tan\left(\frac{\pi}{3}+x\right) + \tan\left(\frac{2\pi}{3}+x\right)$

$= \tan x + \frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x}$

$= \tan x + \frac{8\tan x}{1-3\tan^2 x} = \frac{3(3\tan x - \tan^3 x)}{1-3\tan^2 x} = 3\tan 3x$

Therefore, the given equation is $3\tan 3x = 3$

$\Rightarrow \tan 3x = 1.$

Q.22 (4)

$\sin 47^o + \sin 61^o - (\sin 11^o + \sin 25^o)$

$= 2\sin 54^o\cos 7^o - 2\sin 18^o\cos 7^o$

$= 2\cos 7^o(\sin 54^o - \sin 18^o)$

$= 2\cos 7^o . 2\cos 36^o . \sin 18^o$

$= 4.\cos 7^o.\frac{\sqrt{5}+1}{4}.\frac{\sqrt{5}-1}{4} = \cos 7^o.$

Q.23 (4)

$\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7} = \left[\frac{\sin\left(2^3.\frac{\pi}{7}\right)}{2^3\sin\left(\frac{\pi}{7}\right)}\right] = \frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = -\frac{1}{8}.$

Q.24 (3)

L.H.S. $= \frac{1}{2}\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A\sin B\sin C} = 2$.

Q.25 (1)

We know that $A + C = 180°$, since $ABCD$ is a cyclic quadrilateral. $\Rightarrow A = 180° - C$

$\Rightarrow \cos A = \cos(180° - C) = -\cos C$

$\Rightarrow \cos A + \cos C = 0$ .....(i)

Now $B + D = 180°$, then $\cos B + \cos D = 0$ ....(ii)

Subtracting (ii) from (i), we get

$\cos A - \cos B + \cos C - \cos D = 0$.

Q.26 (3)

L.H.S.

$= 2\cos(A+B)\cos(A-B) + (2\cos^2 C - 1)$

$= -1 - 2\cos C\cos(A-B) + 2\cos^2 C$

$= -1 - 2\cos C[\cos(A-B) + \cos(A+B)]$

$= -1 - 4\cos A\cos B\cos C$

Q.27 (4)

$\sin 2A + \sin 2B - \sin 2C =$

$2\sin A\cos A + 2\cos(B+C)\sin(B-C)$

$\{\because A + B + C = \pi, \therefore B + C = \pi - A, \cos(B+C) = \cos(\pi - A),$

$\cos(B+C) = -\cos A, \sin(B+C) = \sin A\}$

$= 2\cos A\,[\sin A - \sin(B-C)]$

$= 2\cos A[\sin(B+C) - \sin(B-C)]$

$= 2\cos A.2\cos B.\sin C = 4\cos A.\cos B.\sin C.$

Q.28 (4)

Maximum value of $f(x) = \sqrt{1^2 + 1^2} = \sqrt{2}$

Q.29 (2)

Maximum distance $= \sqrt{(\sqrt{3})^2 + (1)^2} = 2$.

Hence, in the graph of function $\sqrt{3}\sin x + \cos x$, maximum distance of a point from *x*-axis is 2.

Q.30 (1)

$\sqrt{2}\cos\left(x + \frac{\pi}{6} - \frac{\pi}{4}\right) = \sqrt{2}\cos\left(x - \frac{\pi}{12}\right).$

Hence maximum value will be at $x = \frac{\pi}{12}$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

$\tan\alpha + \cot\alpha = a$

By squaring both side, we get

$\Rightarrow \tan^2\alpha + \cot^2\alpha = a^2 - 2$

By squaring both sides again, we get

$\Rightarrow \tan^4 a + \cot^4 a + 2 = a^4 - 4a^2 + 4$

$\Rightarrow \tan^4\alpha + \cot^4\alpha = a^4 - 4a^2 + 2$

Q.2 (1)

$a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$

By squaring both side and adding, we get

$a^2 + b^2 = 3^2 + 4^2 = 25$

Q.3 (4)

$3\tan A + 4 = 0 \Rightarrow \tan A = -\frac{4}{3}$

$\Rightarrow \sin A = \pm\frac{\tan A}{\sqrt{1+\tan^2 A}} = \pm\frac{-4/3}{\sqrt{1+16/9}} = \frac{4}{5}$

(Q A is in 2nd quadrant)

and $\cos A = -\frac{3}{5}$, Thus, $2\cot A - 5\cos A + \sin A$

$= 2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5} = \frac{23}{10}$

Q.4 (4)

$\cos 0 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$

$= 1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\left(\pi - \frac{3\pi}{7}\right) + \cos\left(\pi - \frac{2\pi}{7}\right) + \cos\left(\pi - \frac{\pi}{7}\right)$

$= 1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7} = 1$

Q.5 (4)

$$\frac{\tan\left(x - \frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2} + x\right) - \sin^3\left(\frac{7\pi}{2} - x\right)}{\cos\left(x - \frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2} + x\right)}$$

$$= \frac{(-\cot x)(\sin x) - (-\cos^3 x)}{\sin x(-\cot x)}$$

$$= \frac{-\cos x + \cos^3 x}{-\cos x} = \frac{-\cos x(1 - \cos^2 x)}{-\cos x} = \sin^2 x$$

Q.6 (4)

$\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$

$= \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{\pi}{8}$

$= 2\left(\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}\right) = 2 \times 1 = 2.$

Q.7 (2)

Given expression

$= 3\,[\cos^4\alpha + \sin^4\alpha] - 2\,[\cos^6\alpha + \sin^6\alpha]$

$= 3[1 + 2\cos^2\alpha\sin^2\alpha] - 2[1 + 3\cos^2\alpha\sin^2\alpha]$

$= 3 + 6\cos^2\alpha\sin^2\alpha - 2 - 6\cos^2\alpha\sin^2\alpha = 1$

Q.8 (1)

$$\frac{\tan(180° - 25°) - \tan(90° + 25°)}{1 + (\tan(180° - 25°)\tan(90° + 25°)}$$

$$= \frac{-\tan 25° + \frac{1}{\tan 25°}}{2} = \frac{1 - x^2}{2x}$$

Q.9 (1)

$$\frac{\sin 24^o \cos 6^o - \sin 6^o \cos 4^o}{\sin 21^o \cos 39^o - \sin 39^o \cos 21^o}$$

$$= \frac{\sin(24^o - 6^o)}{\sin(21^o - 39^o)} = \frac{\sin 18^o}{\sin(-18^o)} = -1$$

Q.10 (4)

$\because$ 3 sinα = 5 sinβ

$$\Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{5}{3}$$

$$\Rightarrow \frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{8}{2} \Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

Q.11 (1)

tan A + tan B = a

tan A tan B = b

$$\Rightarrow \tan(A+B) = \frac{a}{1-b}$$

$$\therefore \sin^2(A+B) = \left[\frac{|a|}{\sqrt{a^2 + (1-b)^2}}\right]^2$$

$$= \frac{a^2}{a^2 + (1-b)^2}$$

Q.12 (2)

$\because$ tan A < 0 and A + B + C = 180°

$\Rightarrow$ A > 90° $\Rightarrow$ B + C < 90°

$$\Rightarrow \tan(B+C) > 0 \quad \Rightarrow \frac{\tan B + \tan C}{1 - \tan B \tan C} > 0$$

$\Rightarrow$ 1 – tan B tan C > 0 $\Rightarrow$ tan B tan C < 1

Q.13 (3)

tanA – tanB = x

cotB – cotA = y

$$\frac{\tan A - \tan B}{\tan A \tan B} = y$$

$$\Rightarrow \tan A \tan B = \frac{x}{y}$$

$$\text{Now } \cot(A-B) = \frac{1}{\tan(A-B)}$$

$$= \frac{1 + \tan A \tan B}{\tan A - \tan B} = \frac{1 + \frac{x}{y}}{x} = \frac{1}{x} + \frac{1}{y}$$

Q.14 (2)

$\because$ cot (A + B) = cot 225° = 1

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$$

$\Rightarrow$ cot A cot B = 1 + cot A + cot B

$$\text{Now } \frac{\cot A . \cot B}{1 + \cot A + \cot B + \cot A \cot B}$$

$$= \frac{1 + \cot A + \cot B}{2(1 + \cot A + \cot B)} = \frac{1}{2}$$

Q.15 (4)

sinα sin β – cosα cos β + 1 = 0

$\Rightarrow$ – cos (α + β) + 1 = 0 $\Rightarrow$ cos (α + β) = 1

$\Rightarrow$ sin (α + β) = 0

$$\text{then } 1 + \cot\alpha \tan\beta = 1 + \frac{\cos\alpha \sin\beta}{\sin\alpha \cos\beta}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\sin(\alpha+\beta)}{\sin\alpha \cos\beta} = 0$$

Q.16 (2)

$$= \frac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$$

$$= \frac{2\cos 5x \cos x + 10\cos 3x \cos x + 20\cos^2 x}{\cos 5x + 5\cos 3x + 10\cos x}$$

$$= \frac{2\cos x[\cos 5x + 5\cos 3x + 10\cos x]}{[\cos 5x + 5\cos 3x + 10\cos x]} = 2\cos x$$

Q.17 (3)

$$\text{Given, } \cos(A-B) = \frac{3}{5}$$

$$\Rightarrow \cos A \cos B + \sin A \sin B = \frac{3}{5} \quad ....(i)$$

Also, tan A tan B = 2

$\Rightarrow$ 2 cos A cos B – sin A sin B = 0 ....(ii)

From (i) and (ii), we get

$$\cos A \cos B = \frac{1}{5} \ \& \ \sin A \sin B = \frac{2}{5}$$

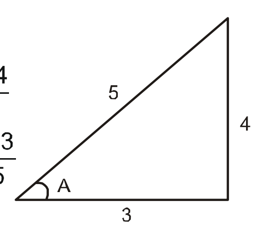
$$\therefore \cos A \cos B - \sin A \sin B = \frac{1}{5} - \frac{2}{5}$$

$$\Rightarrow \cos(A+B) = -\frac{1}{5}$$

Q.18 (1)

$$\sin A = \frac{-4}{5}$$

$$\cos A = \frac{-3}{5}$$



$\tan A = \frac{4}{3} \Rightarrow A \rightarrow$ IIIrd quadrant

$5 \sin 2A + 3 \sin A + 4 \cos A$
$= 10 \sin A \cos A + 3 \sin A + 4 \cos A$
$= 10 \sin A \cos A + 3 \sin A + 4 \cos A$

$= 0$

$= 10 \times \frac{-4}{5} \times \frac{-3}{5} + 3 \times \frac{-4}{5} + 4 \times \frac{-3}{5}$

$= \frac{+120}{25} - \frac{12}{5} - \frac{12}{5} = 0$

Q.19 (1)

$\alpha \in \left[\frac{\pi}{2}, \pi\right]$, $y = \sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha}$

$\Rightarrow y^2 = 1 + \sin\alpha + 1 - \sin\alpha$
$- 2\sqrt{1+\sin\alpha}\,\sqrt{1-\sin\alpha}$

$\Rightarrow y^2 = 2 - 2\sqrt{1-\sin^2\alpha}$ $\because \{\sqrt{1-\sin^2\alpha} = -\cos\alpha\}$

$\Rightarrow y^2 = 2(1 + \cos\alpha)$; $\alpha \in \left[\frac{\pi}{2}, \pi\right]$

$\Rightarrow y^2 = 2 . 2\cos^2\frac{\alpha}{2}$; $\frac{\alpha}{2} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\Rightarrow y = \pm 2\cos\frac{\alpha}{2} \Rightarrow y = 2\cos\frac{\alpha}{2}$

Q.20 (1)

We have, $\sin\alpha + \sin\beta = \frac{-21}{65}$

& $\cos\alpha + \cos\beta = \frac{-27}{65}$

Squaring and adding both sides, we get

$\Rightarrow 2 + 2\cos(\alpha - \beta) = \frac{21^2 + 27^2}{65^2}$

$\Rightarrow 2(1 + \cos(\alpha - \beta)) = \frac{21^2 + 27^2}{65^2}$

$\Rightarrow 2 . 2\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{3^2(7^2+9^2)}{65^2}$

$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{3\sqrt{130}}{130} = \pm\frac{3}{\sqrt{130}}$

$\because \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2}$ $\therefore \cos\left(\frac{\alpha-\beta}{2}\right) < 0$

Hence $\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$

Q.21 (2)

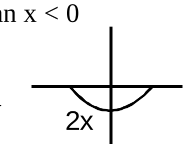
$0 < x < \pi$ & $\cos x + \sin x = \frac{1}{2}$

By squaring both sides, we get

$\Rightarrow 1 + \sin 2x = \frac{1}{4} \Rightarrow \sin 2x = -\frac{3}{4}$

$\because \pi < 2x < 2\pi \Rightarrow \frac{\pi}{2} < x < \pi$ $\therefore \tan x < 0$

Now, $\sin 2x = \frac{-3}{4} \Rightarrow \frac{2\tan x}{1 + \tan^2 x} = -\frac{3}{4}$



$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\Rightarrow \tan x = \frac{-8 \pm \sqrt{28}}{6}$

$\Rightarrow \tan x = \frac{-4 \pm \sqrt{7}}{3}$, $\tan x < 0$

Q.22 (3)

$\cos A = \frac{3}{4}$

$16\cos^2\frac{A}{2} - 32\sin\frac{A}{2}\sin\frac{5A}{2}$

$= \frac{16(1+\cos A)}{2} - 16(\cos 2A - \cos 3A)$

$= \frac{16(1+\cos A)}{2} - 16\ \{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\}$

$= 8\left(1+\frac{3}{4}\right) - 16\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = 3$

Q.23 (2)

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta = \cos\theta(4\cos^2\theta - 3)$

$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left\{4\times\frac{1}{4}\left(a+\frac{1}{a}\right)^2 - 3\right\}$

$= \frac{1}{2}\left(a+\frac{1}{a}\right)\left\{a^2 + \frac{1}{a^2} - 1\right\} = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

Q.24 (3)

$\sin t + \cos t = \frac{1}{5}$

$\Rightarrow \frac{2\tan\frac{t}{2} + 1 - \tan^2\frac{t}{2}}{1 + \tan^2\frac{t}{2}} = \frac{1}{5}$

$\Rightarrow 10\tan^2\frac{t}{2} + 5 - 5\tan^2\frac{t}{2} = 1 + \tan^2\frac{t}{2}$

$\Rightarrow 6\tan^2\frac{t}{2} - 10\tan\frac{t}{2} - 4 = 0$

$\Rightarrow 3\tan^2\frac{t}{2} - 6\tan\frac{t}{2} + \tan\frac{t}{2} - 2 = 0$

$\Rightarrow 3\tan\frac{t}{2}\left(\tan\frac{t}{2}-2\right) + 1\left(\tan\frac{t}{2}-2\right) = 0$

$\Rightarrow \tan\frac{t}{2} = 2,\ \tan\frac{t}{2} = -\frac{1}{3}$

Q.25 (4)

cot x + cot (60º + x) + cot (120º + x)

$= \frac{1}{\tan x} + \tan(30^\circ - x) - \tan(30^\circ + x)$

$= \frac{1}{\tan x} + \left(\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}\right) - \left(\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}\right)$

$= \frac{3-\tan^2 x+\sqrt{3}\tan x-3\tan^2 x-\tan^2 x+\sqrt{3}\tan^2 x-\sqrt{3}\tan^2 x-3\tan^2 x-\tan^2 x}{\tan x(3-\tan^2 x)}$

$= \frac{3-9\tan^2 x}{3\tan x-\tan^3 x}$

Q.26 (2)

$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$

$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$= \sin^2\frac{\pi}{10}\ .\ \sin^2\frac{3\pi}{10}$

$= \left(\frac{\sqrt{5}-1}{4}\cdot\frac{\sqrt{5}+1}{4}\right)^2 = \left(\frac{4}{16}\right)^2 = \frac{1}{16}$

Q.27 (4)

sin12º. sin48º sin54º

Multiplying & dividing by 2, we get

$= \frac{1}{2}(2\sin 12^\circ \sin 48^\circ)\sin 54^\circ$

$= \frac{1}{2}[\cos 36^\circ - \cos 60^\circ]\sin 54^\circ$

$= \frac{1}{2}\left[\cos 36^\circ - \frac{1}{2}\right]\sin 94^\circ$

$= \frac{1}{4}[2\cos 36^\circ \sin 54^\circ - \sin 54^\circ]$

$= \frac{1}{4}[\sin 90^\circ + \sin 18^\circ - \sin 54^\circ]$

$= \frac{1}{4}\left[1+\frac{\sqrt{5}-1}{4}-\frac{\sqrt{5}+1}{4}\right] = \frac{1}{4}\left[1-\frac{1}{2}\right] = \frac{1}{8}$

Q.28 (3)

A = tan 6° tan 42°

B = cot 66° cot 78°

$\frac{A}{B}$ = tan 6° tan 42° tan 66° tan 78°

$\Rightarrow \frac{A}{B} = \frac{\tan 6^\circ \tan(60^\circ-6^\circ)\tan(60^\circ+6^\circ)}{\tan 54^\circ}$.

tan 78° tan 42°

$\Rightarrow \frac{A}{B} = \frac{\tan 18^\circ.\ \tan(60^\circ-18^\circ)\tan(60^\circ+18^\circ)}{\tan 54^\circ}$

$= \frac{\tan 54^\circ}{\tan 54^\circ}$

$\Rightarrow \frac{A}{B} = 1 \Rightarrow A = B$

Q.29 (4)

$\cos\frac{\pi}{10}\cdot\cos\frac{2\pi}{10}\cdot\cos\frac{4\pi}{10}\cdot\cos\frac{8\pi}{10}\cdot\cos\frac{16\pi}{10}$

$= \frac{\sin 2^5\frac{\pi}{10}}{2^5\sin\frac{\pi}{10}} = \frac{1}{32}\frac{\sin\frac{32\pi}{10}}{\sin\frac{\pi}{10}} = \frac{1}{32}\frac{\sin\left(3\pi+\frac{2\pi}{10}\right)}{\sin\left(\frac{\pi}{10}\right)}$

$= -\frac{1}{32}\cdot\frac{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}} = -\frac{1}{16}\cos\frac{\pi}{10}$

$= -\frac{1}{64}\sqrt{10+2\sqrt{5}}$

Q.30 (1)

$\because \alpha+\beta+\gamma = 2\pi \Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \pi$

$\Rightarrow \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

Q.31 (4)

If $A + B + C = \frac{3\pi}{2}$ then

2 cos (A + B) cos (A – B) + cos 2C

= – 2 sin C cos (A – B) + cos2C

$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C$

= 1 – 2sinC [cos (A – B) + sin C]

= –2 sin C [cos(A – B) + sin [3π/2 – (A + B)]]

= 1 – 2 sin C [cos (A – B) – cos (A + B)]

= 1 – 2 sin C . 2 sin A sin B

$= 1 - 4 \sin A \sin B \sin C$

Q.32 (3)

$f(\theta) = \sin^4\theta + \cos^2\theta$

$= \sin^2\theta\,(1 - \cos^2\theta) + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta - \sin^2\theta \cos^2\theta$

$f(\theta) = 1 - \frac{1}{4}\sin^2 2\theta$

$\because\ 0 \le \sin^2 2\theta \le 1$

$f(\theta)_{max} = 1$

$f(\theta)_{min} = 1 - \frac{1}{4} = 3/4$

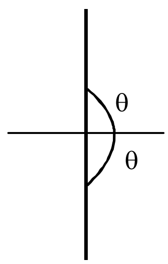
$\therefore$ Range is $\left[\frac{3}{4}, 1\right]$

Q.33 (1)

$-\frac{\pi}{2} < \theta < \frac{\pi}{2};\ \frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$

$= \frac{\sin\theta + 2\sin\theta\cos\theta}{\cos\theta + (1 + \cos 2\theta)} = \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta + 2\cos^2\theta}$

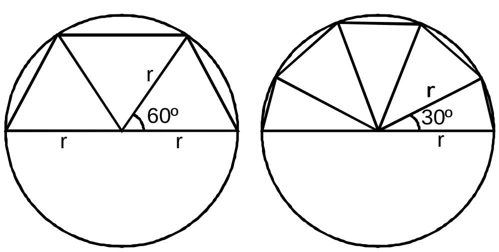
$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)}$ $\left(\because \cos\theta \ne \frac{-1}{2}\right)$



$\Rightarrow \tan\theta \in (-\infty, \infty)$ for $\forall \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Q.34 (4)

Each side of hexagon (inscribe the circle)



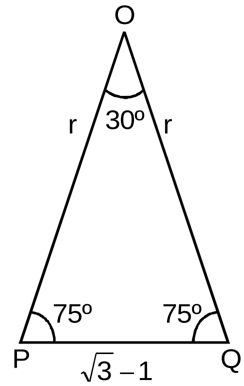
is equal to radius of circle

each side of dodecagon subtends

an angle of 30° at centre of circle

and other two angles are 75º & 75º $\therefore$ PQ = OP cos 75º+OQ cos 75º

$\sqrt{3} - 1 = r\cos 75^\circ + r\cos 75^\circ$



$\Rightarrow 2r\,\frac{(\sqrt{3}-1)}{2\sqrt{2}} = (\sqrt{3} - 1) \Rightarrow r = \sqrt{2}$

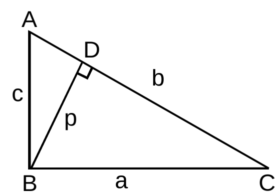
Which is the side of hexagon

Q.35 (2)

Hypotenuse is $2\sqrt{2}$ times of BD

$AC = 2\sqrt{2}\,BD$

$b = 2\sqrt{2}\,p$



In ΔBCD, $\frac{p}{a} = \sin C$ ...(i)

In ΔABC, $\frac{a}{b} = \cos C$ ...(ii)

$\therefore \sin C \cos C = \frac{p}{a} \times \frac{a}{b} = \frac{p}{b}$

$\Rightarrow \sin C \cos C = \frac{1}{2\sqrt{2}} \Rightarrow \sin 2C = \frac{1}{\sqrt{2}}$

$\Rightarrow 2C = \frac{\pi}{4} \Rightarrow C = \frac{\pi}{8},\ A = \frac{\pi}{2} - \frac{\pi}{8} \Rightarrow A = \frac{3\pi}{8}$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

$\operatorname{cosec} A + \cot A = \frac{11}{2}$

$\operatorname{cosec} A - \cot A = \frac{2}{11}$

$\Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22}$

$\Rightarrow \ \cot A = \dfrac{44}{117}$

Q.2 (A)

$2\cos x - 1 = \sin x$

by squaring both sides, we get

$4\cos^2 x - 4\cos x + 1 = \sin^2 x$

$\Rightarrow 5\cos^2 x - 4\cos x = 0$

$\Rightarrow \cos x\,(5\cos x - 4) = 0$

$\therefore \cos x = 0$ or $\cos x = \dfrac{4}{5}$

Given $(\sin x + \cos x) = 1 - \cos x$

Now $6(\sin x + \cos x) + \cos x = 6(1 - \cos x) + \cos x$

$= 6 - 5\cos x = 6 - 5(0) = 6$

or $= 6 - 4 = 2$

Q.3 (D)

$\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}$ ...(i) &

$\dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$ ...(ii)

Multiplying (i) & (ii), we get

$\dfrac{\sin A}{\sin B} \times \dfrac{\cos A}{\cos B} = \dfrac{\sqrt{3}}{2} \times \dfrac{2}{\sqrt{5}} \Rightarrow \dfrac{\tan A}{\tan B} = \dfrac{\sqrt{3}}{\sqrt{5}}$

Applying componendo, we get

$\Rightarrow \dfrac{\tan A + \tan B}{\tan B} = \dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ ...(iii)

Now from (i) & (ii), $4\sin^2 A = 3\sin^2 B$ ....(iv)

& $4\cos^2 A = 5\cos^2 B$ ...(v)

By adding (iv) & (v), $4 = 3 + 2\cos^2 B$

$\Rightarrow \cos^2 B = \dfrac{1}{2} \Rightarrow \sec^2 B = 2$

$\Rightarrow \tan^2 B = 1 \Rightarrow \tan B = 1$

Put these values in (iii), $\tan A + \tan B = \dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$

Q.4 (A)

$3\sin x + 4\cos x = 5$

By squaring both sides,

$9\sin^2 x + 16\cos^2 x = 25 - 24\sin x\cos x$

$\Rightarrow 9(1-\cos^2 x) + 16(1-\sin^2 x) = 25 - 24\sin x\cos x$

$\Rightarrow 9 + 16 - 25 = 9\cos^2 x + 16\sin^2 x - 24\sin x\cos x$

$\Rightarrow 0 = (4\sin x - 3\cos x)^2 \quad \Rightarrow 4\sin x - 3\cos x = 0$

Q.5 (C)

$a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ ... (i)

$a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$ ... (ii)

then $(m+n)^{2/3} + (m-n)^{2/3}$

$= (a(\cos^3\alpha + \sin^3\alpha) + 3a\cos\alpha\sin\alpha(\cos\alpha + \sin\alpha))^{2/3} + (a(\cos^3\alpha - \sin^3\alpha) + 3a\cos\alpha\sin\alpha(\sin\alpha - \cos\alpha))^{2/3}$

$= a^{2/3}[\{\cos^3\alpha + \sin^3\alpha + 3\cos\alpha\sin\alpha(\cos\alpha + \sin\alpha)\}^{2/3} + \{\cos^3\alpha - \sin^3\alpha - 3\cos\alpha\sin\alpha(\cos\alpha - \sin\alpha)\}^{2/3}]$

$= a^{2/3}[\{(\cos\alpha + \sin\alpha)^3\}^{2/3} + \{(\cos\alpha - \sin\alpha)^3\}^{2/3}]$

$= a^{2/3}[(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2] = a^{2/3}[1 + 1] = 2a^{2/3}$

Q.6 (C)

We have,

$0° < x < 90°$; $\cos x = \dfrac{3}{\sqrt{10}}$, then

$\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}\sin x\cos x\tan x$

$\left\{\sin^2 x = 1 - \dfrac{9}{10} = \dfrac{1}{10}\right\} = \log_{10}\sin^2 x = \log_{10}\left(\dfrac{1}{10}\right) = -1$

Q.7 (A)

Given that,

$\cot\alpha + \tan\alpha + = m$ & $\dfrac{1}{\cos\alpha} - \cos\alpha = n$

$\Rightarrow \dfrac{\cos\alpha}{\sin\alpha} + \dfrac{\sin\alpha}{\cos\alpha} = m$ & $\dfrac{\sin^2\alpha}{\cos\alpha} = n$...(1)

$\Rightarrow \dfrac{1}{m} = \sin x\cos x$ ...(2)

From (1) & (2)

$\cos\theta = \dfrac{1}{m}\left(\dfrac{m}{n}\right)^{\frac{1}{3}}$ & $\sin\theta = \left(\dfrac{n}{m}\right)^{\frac{1}{3}}$

$\Rightarrow \left[\dfrac{1}{m}\left(\dfrac{m}{n}\right)^{\frac{1}{3}}\right]^2 + \left[\left(\dfrac{n}{m}\right)^{\frac{1}{3}}\right]^2 = 1$

$\Rightarrow \dfrac{1}{m^2}\left(\dfrac{m}{n}\right)^{\frac{2}{3}} + \left(\dfrac{n}{m}\right)^{\frac{2}{3}} = 1$

$\Rightarrow 1 + m^{2/3}n^{4/3} = m^{4/3}n^{2/3}$

$\Rightarrow m^{4/3}n^{2/3} - m^{2/3}n^{4/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - n(m^2n)^{1/3} = 1$

Q.8 (A)

$2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = \dfrac{15}{4}$

$\Rightarrow -\sec^4\alpha + 2\sec^2\alpha - 1 + 1 + \operatorname{cosec}^4\alpha - 2\operatorname{cosec}^2\alpha =$

$\frac{15}{4}$

$\Rightarrow -(\sec^2\alpha - 2\sec^2\alpha + 1) + (\text{cosec}^4\alpha - 2\cos^2\alpha + 1)$

$= \frac{15}{4}$

$\Rightarrow -(\sec^2\alpha - 1)^2 + (\text{cosec}^2\alpha - 1)^2 = \frac{15}{4}$

$\Rightarrow -\tan^4 a + \cot^4\alpha = \frac{15}{4}$

$\Rightarrow -4\tan^8\alpha + 4 = 15\tan^4\alpha$

$\Rightarrow 4\tan^8\alpha + 15\tan^4\alpha - 4 = 0$

$\Rightarrow (\tan^4\alpha + 4)(4\tan^4\alpha - 1) = 0$

$\Rightarrow \tan^4\alpha = -4$ or $\tan^4\alpha = \frac{1}{4}$

$\Rightarrow \tan^2\alpha = \pm\frac{1}{2} \Rightarrow \tan\alpha = \pm\frac{1}{\sqrt{2}}$

Q.9 (B)

$$\frac{\cos^2\theta\sin^3\theta\tan^5\theta}{(-\sin^3\theta)\cos^2\theta(-\tan^5\theta)} = 1$$

Q.10 (B)

$-\sin(810° + 60°) - \text{cosec}(720° - 60°) - \tan(720° + 135°) + 2\cot(720° + 120°) + \cos(360° + 120°)$

$+ \sec(720° + 180°)$

$-\sin(810° + 60°) - \text{cosec}(720° - 60°) - \tan(135°) + 2\cot(120°) + \cos(120°) + \sec(180°)$

$= -\frac{1}{2} + \frac{2}{\sqrt{3}} + 1 - \frac{2}{\sqrt{3}} - \frac{1}{2} - 1 = -1.$

$\therefore$ Absolute value $= |-1| = 1.$

Q.11 (B)

$$\frac{1}{\cos 290^o} + \frac{1}{\sqrt{3}\sin 250^o}$$

$$= \frac{1}{\cos(270° + 20°)} + \frac{1}{\sqrt{3}\sin(270° - 20°)}$$

$$= \frac{1}{\sin 20^o} - \frac{1}{\sqrt{3}\cos 20^o} = \frac{(\sqrt{3}\cos 20^o - \sin 20^o)}{(\sqrt{3}\sin 20^o\cos 20^o)}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^o - \frac{1}{2}\sin 20^o\right)}{\sqrt{3}\sin 20^o\cos 20^o} = \frac{2\sin(60^o - 20^o)}{\left(\frac{\sqrt{3}}{2}\right)\sin 40^o}$$

$$= \frac{4\sin 40^o}{\sqrt{3}\sin 40^o} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

Q.12 **B**

Given that,

$f(\theta) = \sin^2\theta + \sin^2\left(\theta + \frac{2\pi}{3}\right) + \sin^2\left(\theta + \frac{4\pi}{3}\right)$

$= 1 + \sin^2\theta - [\cos^2\left(\theta + \frac{2\pi}{3}\right) - \sin^2\theta\left(\theta + \frac{\pi}{3}\right)]$

$= 1 + \sin^2\theta - \cos(2\theta + \pi)\cos\frac{\pi}{3}$

$= 1 + \sin^2\theta + \frac{\cos 2\theta}{2}$

$= 1 + \sin^2\theta + \frac{1}{2} - \sin^2\theta = \frac{3}{2}$

Hence, $f\left(\frac{\pi}{15}\right) = \frac{3}{2}$

Q.13 (A)

we have, 3 cos + 2 cos 3x = cos y

and 3 sin x + 2 sin 3x = sin y

Squaring and adding both sides, we get

$(3\cos x + 2\cos 3x)^2 + (3\sin x + 2\sin 3x)^2$

$= \cos^2 y + \sin^2 y$

$\Rightarrow 9 + 4 + 12\cos x\cos 3x + 12\sin x\sin 3x = 1$

$\Rightarrow 12\cos(3x - x) = 12 \Rightarrow \cos 2x = -1$

Q.14 (B)

$$\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}} = \sqrt{\frac{2\cos\alpha}{\sin\alpha} + \frac{1}{\sin^2\alpha}}$$

$$= \sqrt{\frac{2\sin\alpha\cos\alpha + 1}{\sin^2\alpha}} = \sqrt{\frac{(\sin\alpha + \cos\alpha)^2}{\sin^2\alpha}}$$

$$= \frac{|(\sin\alpha + \cos\alpha)|}{\sin\alpha} \quad \begin{cases}\text{In given Interval} \\ |\sin\alpha| < |\cos\alpha| \\ \& \sin\alpha + \cos\alpha < 0\end{cases}$$

$$= -\frac{(\sin\alpha + \cos\alpha)}{\sin\alpha} = -(1 + \cot\alpha) = -1 - 1\cot\alpha$$

Q.15 (B)

Given expression

$= \tan\frac{\pi}{16} + 2\tan\frac{\pi}{8} + 4$

$= \tan\frac{\pi}{16} + 2(\sqrt{2} - 1) + 4$

(Let $\pi/16 = \theta \Rightarrow \pi = 16\theta \Rightarrow \pi/2 = 8\theta$)

$= \tan\theta + 2(\sqrt{2} + 1)$

$= \tan\theta + 2\cot 2\theta$

$= \tan\theta + \dfrac{2(1-\tan^2\theta)}{2\tan\theta}$

$= \dfrac{\tan^2\theta + 1 - \tan^2\theta}{\tan\theta} = \cot\theta = \cot\dfrac{\pi}{16}$

Q.16 (B)

Given that, $\cos\alpha + \cos\beta = a$ & $\sin\alpha + \sin\beta = b$, & $\alpha - \beta = 2\theta$

Squaring & adding both sides,

$(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta)$

$+ 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= a^2 + b^2$

$\Rightarrow 1 + 1 + 2\cos(\alpha - \beta) = a^2 + b^2 \quad \Rightarrow 2 + 2\cos(2\theta) = a^2 + b^2$

$\Rightarrow 2(1 + \cos 2\theta) = a^2 + b^2$

$\Rightarrow 4\cos^2\theta = a^2 + b^2$

Now $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\therefore \dfrac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3 = a^2 + b^2 - 3$

Q.17 (A)

$\cos A\cos B\cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$

$\Rightarrow \cos A\cos B\cos C = \lambda(4\cos^3 A - 3\cos A + 4\cos^3 B - 3\cos B + 4\cos^3 C - 3\cos C)$

$\Rightarrow \cos A\cos B\cos C = \lambda(4(\cos^3 A + \cos^3 B + \cos^3 C) - 3\times 0)$

$\because \cos A + \cos B + \cos C = 0$

$\Rightarrow \cos^3 A + \cos^3 B + \cos^3 C = 3\cos A\cos B\cos C$

$\Rightarrow \cos A\cos B\cos C = 12\lambda\cos A\cos B\cos C \Rightarrow \lambda = \dfrac{1}{12}$

Q.18 (B)

Given that, $\sin 2\theta = k$

$\dfrac{\tan^3\theta}{(1+\tan^2\theta)} + \dfrac{\cot^3\theta}{(1+\cot^2\theta)}$

$= \dfrac{\sin^3\theta}{\cos^3\theta}\cos^2\theta + \dfrac{\cos^3\theta}{\sin^3\theta}\sin^2\theta$

$= \dfrac{2(\sin^4\theta + \cos^4\theta)}{2(\sin\theta\cos\theta)}$

$= \dfrac{2[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]}{\sin 2\theta}$

$= \dfrac{2\left[1 - \frac{1}{2}\sin^2 2\theta\right]}{\sin 2\theta} = \dfrac{2 - \sin^2 2\theta}{\sin 2\theta} = \dfrac{2-k^2}{k}$

Q.19 (A)

$\cos\dfrac{\pi}{19} + \cos\dfrac{3\pi}{19} + \cos\dfrac{5\pi}{19} + \ldots + \cos\dfrac{17\pi}{19}$

$= \dfrac{2\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}}\left[\cos\dfrac{\pi}{19} + \cos\dfrac{3\pi}{19} + \cos\dfrac{5\pi}{19} + \ldots + \cos\dfrac{17\pi}{19}\right]$

$= \dfrac{1}{2\sin\frac{\pi}{19}}\left[\sin\frac{2\pi}{19} + \left(\sin\frac{4\pi}{19} - \sin\frac{2\pi}{19}\right) + \left(\sin\frac{6\pi}{19} - \sin\frac{4\pi}{19}\right) + \ldots + \left(\sin\frac{18\pi}{19} - \sin\frac{16\pi}{19}\right)\right]$

$= \dfrac{\sin\frac{18\pi}{19}}{2\sin\frac{\pi}{19}} = \dfrac{\sin\left(\pi - \frac{\pi}{19}\right)}{2\sin\frac{\pi}{19}} = \dfrac{\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}} = \dfrac{1}{2}$

Aliter : Use sum of cosine series

Q.20 (A)

Given $A + B + C = \pi$

Now, $\dfrac{\sin\left(A + \frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)} = k$

By applying componendo & dividendo,

$\Rightarrow \dfrac{\sin\left(A + \frac{C}{2}\right) - \sin\left(\frac{C}{2}\right)}{\sin\left(A + \frac{C}{2}\right) + \sin\left(\frac{C}{2}\right)} = \dfrac{k-1}{k+1}$

$\Rightarrow \dfrac{2\cos\left(\frac{A}{2} + \frac{C}{2}\right)\sin\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2} + \frac{C}{2}\right)\cos\left(\frac{A}{2}\right)} = \dfrac{k-1}{k+1}$

$\Rightarrow \dfrac{\tan\frac{A}{2}}{\tan\left(\frac{A}{2} + \frac{C}{2}\right)} = \dfrac{k-1}{k+1} \Rightarrow \dfrac{\tan\frac{A}{2}}{\tan\left(\frac{\pi}{2} - \frac{B}{2}\right)} = \dfrac{k-1}{k+1}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} = \frac{k-1}{k+1}$

Q.21 $p(x) = \sec^2x \cdot \csc^2x = \frac{4}{4\cos^2 x \cdot \sin^2 x}$

$p(x) = \frac{4}{\sin^2 2x}$

$p(x)|_{min.} = 4$

Q.22 (B)

$E = \frac{7+6\tan\theta-\tan^2\theta}{\sec^2\theta} = 7\cos^2\theta + 6\sin\theta\cos\theta$

$-\sin^2\theta$

$\Rightarrow \frac{7}{2}(1+\cos 2\theta) + 3\sin 2\theta - \frac{1}{2}(1-\cos 2\theta)$

$\Rightarrow 3 + 3\sin 2\theta + 4\cos 2\theta = 3 \pm 5$

$\Rightarrow M = 8,\ m = -2$

$\Rightarrow M - m = 10$.

Q.23 (A)

$u^2 = 5+2\sqrt{(u\cos^2\theta-\sin^2\theta)(4\sin^2\theta+\cos^2\theta)}$

$= 5+2\sqrt{(4-3\sin^2\theta)(1+3\sin^2\theta)}$

$= 5+2\sqrt{4+9\sin^2\theta-9\sin^2\theta}$

$= 5+2\sqrt{4-9\left(\sin^4\theta-\sin^2\theta+\frac{1}{4}-\frac{1}{4}\right)}$

$= 5+2\sqrt{4-9\left(\sin^2\theta-\frac{1}{2}\right)^2+\frac{9}{4}}$

$u^2 = 5+2\sqrt{\frac{25}{4}-9\left(\sin^2\theta-\frac{1}{2}\right)^2}$

$\max = 5 + 2\cdot\frac{5}{2} = 10$

$\min = 5 + 2\cdot 2 = 9$

Q.24 (A)

Let $\log_4 A = \log_6 B = \log_9(A+B) = x$

$\Rightarrow A = 4^x;\ B = 6^x$ and $A + B = 9^x$;

we have to find $\frac{B}{A} = \left(\frac{3}{2}\right)^x$

$\Rightarrow 4^x + 6^x = 9^x \quad \Rightarrow 2^{2x} + 2^x\cdot 3^x = 3^{2x}$

let $2^x = a;\ 3^x = b$

$\Rightarrow b^2 - ab - a^2 = 0$

$\frac{b^2}{a^2} - \frac{b}{a} - 1 = 0 \Rightarrow \frac{b}{a} = \frac{1\pm\sqrt{5}}{2}$

$\therefore \frac{b}{a} = \frac{\sqrt{5}+1}{2} \Rightarrow \left(\frac{3}{2}\right)^x = \frac{B}{A} = 2\cos 36°$.

Q.25 (B)

We have $\log_{\cos x}\left(\frac{\sqrt{3}}{2}\sin x\right) - \log_{\cos x}(\tan x) = 2$

or $\log_{\cos x}\frac{\sqrt{3}}{2}\left(\frac{\sin x}{\sin x}\cdot\cos x\right) = 2.$

$\Rightarrow \log_{\cos x}\left(\frac{\sqrt{3}}{2}\cos x\right) = 2$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x = \cos^2 x. \Rightarrow \cos x\left(\cos x - \frac{\sqrt{3}}{2}\right) = 0$

$\Rightarrow \cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$

$\therefore x = \frac{\pi}{6} \qquad \left(x = \frac{\pi}{2}\text{ is rejected}\right)$.

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B, D)

$\frac{\sin x+\cos x}{\cos^3 x} = \tan x\sec^2x + \sec^2x = \sec^2x(1+\tan x)$

$= (1+\tan^2x)(1+\tan x)$

Q.2 (A, B)

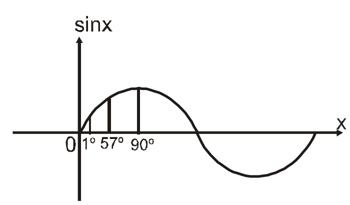
$1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$

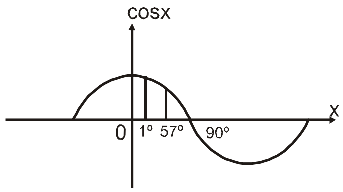
$\Rightarrow (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$

So each side is equal to ± 1

Q.3 (B, C)

1 radian $\simeq$ 57° (approx.)





$\sin 1 > \sin 1°$
$\cos 1° > \cos 1$

Q.4 (A, B, C, D)

$\frac{1+\tan 11^o}{1-\tan 11^o}$ = tan (45º + 11º) = tan 56º

Q.5 (C, D)

$0 < \theta < \pi/2$

$\tan 3\theta = \tan(2\theta + \theta) = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$

$\Rightarrow \tan 3\theta - \tan\theta \tan 2\theta \tan 3\theta = \tan 2\theta + \tan\theta$
$\Rightarrow \tan 3\theta - \tan 2\theta - \tan\theta = \tan\theta \tan 2\theta \tan 3\theta$
$\Rightarrow \tan 3\theta + \tan 3\theta = \tan\theta \,.\, \tan 2\theta \tan 3\theta$
($\because$ given that $\tan\theta + \tan 2\theta + \tan 3\theta = 0$)
$\Rightarrow \tan 3\theta\,(2 - \tan\theta \tan 2\theta) = 0$
$\Rightarrow \tan 3\theta = 0$ or $\tan\theta \tan 2\theta = 2$.

Q.6 (C, D)

$\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$
(Given x + y = z)
$= 1 + \cos(x + y)\cos(x - y) + \cos^2 z - 2\cos x \cos y \cos z$
$= 1 + \cos z\,[\cos(x - y) + \cos(x + y)] - 2\cos x \cos y \cos z$
$= 1 + \cos z \,.\, 2\cos x \cos y - 2\cos x \cos y \cos z$
$= 1$
$= \cos(x + y - z)$

Q.7 (B, C)

$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ & $\tan x = \frac{2b}{a-c}$
$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$
$\Rightarrow y + z = a + c$
and $y - z = (a - c)\left(\cos^2 x - \sin^2 x\right) + 4b\sin x\cos x$
$= (a - c)\cos 2x + 2b\sin 2x$ ($\because 2b = (a - c)\tan x$)
$= (a - c)\,[\cos 2x + \tan x.\sin 2x] = (a - c)\left[2\cos 2x + \frac{\sin x}{\cos x}\sin 2x\right]$

$= \frac{(a-c)\cos(2x-x)}{\cos x} = (a - c).$

Q.8 (B, D)

$$\left\{\frac{2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right\}^n + \left\{\frac{2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right\}^n$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n \cot^n\left(\frac{A-B}{2}\right)$$

0 ; $n \in$ odd
$2\cot^n\left(\frac{A-B}{2}\right)$; $n \in$ even

Q.9 (A,B)

$3\sin\beta = \sin(2\alpha + \beta)$

$\Rightarrow \frac{\sin(2\alpha+\beta)}{\sin\beta} = \frac{3}{1}$

Applying componendo & dividendo

$\Rightarrow \frac{\sin(2\alpha+\beta)+\sin\beta}{\sin(2\alpha+\beta)-\sin\beta} = \frac{3+1}{3-1} = \frac{2}{1}$

$\Rightarrow \frac{2\sin(\alpha+\beta).\cos\alpha}{2\cos(\alpha+\beta)\sin\alpha} = 2$

$\Rightarrow \tan(\alpha + \beta) = 2\tan\alpha$
$\Rightarrow \tan(\alpha + \beta) - 2\tan\alpha = 0$
Hence, independent of α & β both.

Q.10 (C,D)

$0 < \theta < \pi/2$;
$\tan\theta + \tan 2\theta + \tan 3\theta = 0$

clearly $\tan\theta \neq 0$ and $\tan 2\theta \neq 0$ for $0 < \theta < \frac{\pi}{2}$

Now, $\tan 3\theta = \tan(2\theta + \theta)$ $0 < 2\theta < \pi$

$\Rightarrow \tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta}$ $0 < 3\theta < \frac{3\pi}{2}$

Given that $(\tan\theta + \tan 2\theta) + \tan 3\theta = 0$ (by using given condition)
$\Rightarrow \tan 3\theta\,(1 - \tan 2\theta\tan\theta) + \tan 3\theta = 0$
$\Rightarrow \tan 3\theta\,[2 - \tan 2\theta\tan\theta] = 0$
$\Rightarrow \tan 3\theta = 0$ or $\tan 2\theta\tan\theta = 2$

Q.11 (B, D)

Let $\tan\frac{\alpha}{2} = t$

$(a + 2)\,2t + (2a - 1)(1 - t^2) = (2a + 1)(t^2 + 1)$
$\Rightarrow 2at + 4t + 2a - 2at^2 - 1 + t^2 = 2a + 1 + 2at^2 + t^2$
$\Rightarrow 4at^2 - 2t(2 + a) + 2 = 0 \Rightarrow 2at^2 - 2t - at + 1 = 0$
$\Rightarrow 2t(at - 1) - 1(at - 1) = 0 \Rightarrow t = 1/2$, $t = 1/a$

$\Rightarrow$ $\tan\alpha = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

$\Rightarrow$ $\tan\alpha = \dfrac{2\times 1/2}{1-1/4} = 4/3$

or $\tan\alpha = \dfrac{2/a}{1-1/a^2} = \dfrac{2a}{a^2-1}$

Q.12 (B,C)

Given that,

$\tan x = \dfrac{2b}{a-c}$, $a \neq c$

& $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ ....(i)

& $z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$ ....(ii)

By adding (i) & (ii), we get

$(y+z) = a + c$ (option B correct)

Subtracting (i) & (ii)

$(y-z) = (a-c)\cos^2 x - (a-c)\sin^2 x + 4b\sin x\cos x$

$\Rightarrow$ $(y-z) = (a-c)(\cos^2 x - \sin^2 x) + 2b\sin 2x$

$\Rightarrow y - z = (a-c)\left[1 - 2\sin^2 x + \left(\dfrac{2b}{a-c}\right) 2\sin x\cos x\right]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + \tan x \, 2\sin x\cos x]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + 2\sin^2 x]$

$\Rightarrow y - z = a - c$ (option C correct)

Q.13 (B,D)

Given expression

$\cos\dfrac{\pi}{10}\cos\dfrac{2\pi}{10}\cdot\cos\dfrac{4\pi}{10}\cdot\cos\dfrac{8\pi}{10}\cos\dfrac{16\pi}{10}$

$= \dfrac{\sin 2\left(\dfrac{16\pi}{10}\right)}{2^5\sin\left(\dfrac{\pi}{10}\right)} = \dfrac{\sin\dfrac{32\pi}{10}}{2^5\sin\dfrac{\pi}{10}} = \dfrac{-\sin\dfrac{2\pi}{10}}{2^5\sin\dfrac{\pi}{10}} = -$

$\dfrac{-\cos(\pi/10)}{16}$ (B)

Now, $\cos\dfrac{\pi}{10} = \sqrt{1-\sin^2\dfrac{\pi}{10}} = \sqrt{1-\left(\dfrac{\sqrt{5}-1}{4}\right)^2}$

$= \sqrt{\dfrac{16-(6-2\sqrt{5})}{16}} = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

$\Rightarrow$ $\dfrac{-\cos\dfrac{\pi}{10}}{16} = \dfrac{-\sqrt{10+2\sqrt{5}}}{64}$

(D)

Q.14 (A, B)

$\tan A + \tan B + \tan C - \tan A\tan B\tan C = 0$

$\Rightarrow$ $\cos A\cos B\cos C(\tan A + \tan B + \tan C - \tan A\tan B\tan C) = 0$

$\Rightarrow$ $\sin(A + B + C) = 0 \Rightarrow A + B + C = n\pi$, $n \in I$

Q.15 (A, B)

$\tan A + \tan B + \tan C = 6$, $\tan A\tan B = 2$

In any ΔABC,

$\tan A + \tan B + \tan C = \tan A\tan B\tan C$

$\Rightarrow$ $6 = 2\tan C \Rightarrow \tan C = 3$

$\therefore$ $\tan A + \tan B + 3 = 6$

$\Rightarrow$ $\tan A + \tan B = 3$ & $\tan A\tan B = 2$

Now $(\tan A - \tan B)^2 = (\tan A + \tan B)^2 - 4\tan A\tan B = 9 - 8 = 1$

$\Rightarrow$ $\tan A - \tan B = \pm 1$

$\therefore$ $\tan A - \tan B = 1$ or $\tan A - \tan B = -1$

$\tan A + \tan B = 3$ $\quad$ $\tan A + \tan B = 3$

on solving on solving

$\tan A = 2$ $\tan A = 1$

$\tan B = 1$ $\tan B = 2$

Q.16 (A, C)

h =

$\sqrt{\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta)\}^2 + \{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha+\beta)\}^2}$

$\Rightarrow$ $h = [4\cos^2(\alpha+\beta)(\cos(\alpha-\beta)+1)^2 + 4\sin^2(\alpha+\beta)(\cos(\alpha-\beta)+1)^2]^{1/2}$

$\Rightarrow$ $h = [4\{\cos(\alpha-\beta)+1\}^2\{\cos^2(\alpha+\beta)+\sin^2(\alpha+\beta)\}]^{1/2}$

$\Rightarrow$ $h = 2(1 + \cos(\alpha-\beta))$ $\Rightarrow$ $h = 2\times 2\cos^2\left(\dfrac{\alpha-\beta}{2}\right) \Rightarrow h = 4\cos^2\left(\dfrac{\alpha-\beta}{2}\right)$

Q.17 (B,D)

Let $Y = 1 + 4\sin\theta + 3\cos\theta$

$\because$ Max value of $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}$

Max value of $Y = 1 + \sqrt{4^2+3^2} = 6$

$\because$ Min value of $a\sin\theta + b\cos\theta = -\sqrt{a^2+b^2}$

Min value of $Y = 1 - \sqrt{4^2+3^2} = 1 - 5 = -4$

Q.18 (A,D)

Let $Y = 1 + 4\sin\theta + 3\cos\theta$

$z = 3\sin^2\theta + 3\sin 2\theta + 11\cos^2\theta$

$= 3 + 3\sin 2\theta + 8\cos^2\theta = 3 + 4(1+\cos 2\theta) + 3\sin 2\theta$

$= 7 + 3\sin 2\theta + 4\cos 2\theta$

$\Rightarrow$ maximum value $= 7 + \sqrt{3^2+4^2} = 12$,

minimum value $= 7 - \sqrt{3^2+4^2} = 2$

Q.19 (B, D)

$\sin^6 x + \cos^6 x = a^2$

$\Rightarrow$ $(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$

$\Rightarrow$ $(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2 \Rightarrow 1 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow 1 - \frac{3}{4}\sin^2 2x = a^2 \Rightarrow \frac{4(1-a^2)}{3} = \sin^2 2x$

$\Rightarrow 0 \le \frac{4}{3}(1 - a^2) \le 1$

$1 - a^2 \ge 0$ and $4 - 4a^2 \le 3$

$a^2 \le 1$ and $\frac{1}{4} \le a^2$

$-1 \le a \le 1$ and $a \ge \frac{1}{2}$ or $a \le -\frac{1}{2}$

$a \in \left[-1,-\frac{1}{2}\right] \cup \left[\frac{1}{2},1\right]$

Q.20 (A, C)

$f(x) = \cos 9x + \cos(-10)x$

$= \cos 9x + \cos 10x$

$f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = -1$

$f(\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

$f(-\pi) = 0$

$f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{5\pi}{2} = \cos\frac{9\pi}{4}$

$= \cos\left(2\pi + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Comprehenssion # 1 (Q. No. 21 to 23)

Q.21 **(C)**

$\tan A \tan B \tan C = \tan A + \tan B + \tan C = \frac{p}{q}$

Q.22 (B)

$\tan A \tan B + \tan B \tan C + \tan C \tan A$

$= \frac{\sin A \sin B \cos C + \cos A \sin B \sin C + \cos B \sin C \sin A}{\cos A \cos B \cos C}$

$= \frac{\sin B.\sin(A+C) + \cos B \sin C \sin A}{q}$

$= \frac{1 - \cos^2 B + \cos B \sin C \sin A}{q}$

$= \frac{1 + \cos B\,(\sin C \sin A - \cos B)}{q}$

$= \frac{1 + \cos B \cos C \cos A}{q}$

$= \frac{1+q}{q}$

Q.23 (D)

$\tan^3 A + \tan^3 B + \tan^3 C - 3\tan A \tan B \tan C$

$= (\tan A + \tan B + \tan C)\{(\tan A + \tan B + \tan C)^2 - 3(\tan A \tan B + \tan B \tan C + \tan C \tan A)\}$

$= \frac{p}{q}\left[\left(\frac{p}{q}\right)^2 - 3\left(\frac{1+q}{q}\right)\right]$

$\therefore \tan^3 A + \tan^3 B + \tan^3 C$

$= \frac{3p}{q} + \frac{p}{q^3}(p^2 - 3q - 3q^2)$

$= \frac{3pq^2 + p^3 - 3pq - 3pq^2}{q^3} = \frac{p^3 - 3pq}{q^3}$

Comprehension # 2 (Q. No. 24 to 26)

Q.24 (C)

Given $\cos\alpha + \cos\beta = a$

$\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = a$ ..... (i)

and $\sin\alpha + \sin\beta = b$

$\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = b$ ..... (ii)

by (i) & (ii)

$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$

$\Rightarrow \tan\theta = \frac{b}{a}$

$\therefore \sin 2\theta + \cos 2\theta$

$= \frac{2\left(\frac{b}{a}\right)}{1+\frac{b^2}{a^2}} + \frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}} = \frac{2ab}{a^2+b^2} + \frac{a^2-b^2}{a^2+b^2} =$

$\frac{a^2+b^2-2b^2+2ab}{a^2+b^2} = 1 + \frac{2b(a-b)}{a^2+b^2}$

$\therefore n = 2$

Q.25 (A)

$\sin^n A = x$

$\Rightarrow \sin^2 A = x$

$\therefore \sin A \sin 2A \sin 3A \sin 4A$

$= \sin A\,(2\sin A \cos A)\,(3\sin A - 4\sin^3 A)\,(4\sin A \cos A\,(1 - 2\sin^2 A))$

$= 8\sin^4 A\,(1 - \sin^2 A)\,(1 - 2\sin^2 A)\,(3 - 4\sin^2 A)$

If we put $\sin^2 A = x$, then given expression is a polynomial of degree 5 in x.

Q.26 (B)

If p = 5

$\therefore$ sin x + (p – 5), cos x, tan x

$\Rightarrow$ sin x, cos x, tan x are in G.P.

$\therefore \cos^2 x = \sin x \tan x$

$\cos^3 x = \sin^2 x$

$\therefore \cos^3 x = 1 - \cos^2 x$

$\therefore \cos^3 x + \cos^2 x = 1$

taking cube both sides

$\therefore \cos^9 x + \cos^6 x + 3\cos^5 x = 1$

$\Rightarrow \cos^9 x + \cos^6 x + 3\cos^5 x - 1 = 0$

Q.27 (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

(A) sin 420° cos 390° + cos (–660°) sin (–330°)

= sin (360° + 60°) cos (360° + 30°) + cos 660° (–sin 330°)

= sin 60° cos30° – cos (720° – 60°) sin (360° – 30°)

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \cos 60° (-\sin 30°) = \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{2} = 1$

(B) tan 315° cot (–405°) + cot (495°) tan (–585°)

= tan (360° – 45°) (–cot (360° + 45°)) + cot (360° + 135°) (–tan (720° – 135°))

= (–tan 45°) (–cot 45°) + cot 135° tan 135° = 1 + 1

= 2

(C) 37° = 45° – 8°

$\Rightarrow \tan 37° = \frac{1-\tan 8°}{1+\tan 8°} \Rightarrow 1 + \tan 37° = \frac{2}{1+\tan 8°}$

Similarly $1 + \tan 23° = \frac{2}{1+\tan 22°}$

$\therefore \quad \frac{(1+\tan 8°)(1+\tan 37°)}{(1+\tan 22°)(1+\tan 23°)} = 1$

(D) $\left[\frac{\pi}{4}\right] + \left[\frac{1}{3}\sin^2 x\right] = 0 + 0 = 0$

Q.28 (A) $\rightarrow$ (q, s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

(A) $x + \frac{1}{x} = 2\cos\theta \geq 2$ or ≤ -2

$\Rightarrow \cos\theta = 1$ or -1

(B) sin θ + cosec θ = 2

$\because \sin\theta + \frac{1}{\sin\theta} \geq 2$ or ≤ -2

but given that sin θ + cosec θ = 2 $\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

which is possible only when sin θ = 1

$\therefore \sin^{2008}\theta + \text{cosec}^{2008}\theta = \sin^{2008} + \frac{1}{\sin^{2008}} = 1 + 1 = 2$

(C) $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - \frac{1}{2}\sin^2 2\theta$

$\because 0 \leq \sin^2 2\theta \leq 1$

$\therefore \frac{1}{2} \leq 1 - \frac{1}{2}\sin^2 2\theta \leq 1$

$\therefore$ maximum value = 1

(D) $2\sin^2\theta + 3\cos^2\theta = 2\sin^2\theta + 3 - 3\sin^2\theta$

$= 3 - \sin^2\theta$

$\because 0 \leq \sin^2\theta \leq 1$

$\therefore 2 \leq 3 - \sin^2\theta \leq 3$

$\therefore$ least value = 2

NUMERICAL VALUE BASED

Q.1 Find the value of tan 1º tan 2º tan 3º ..... tan 89º is

Ans. 1

Sol. tan 1º tan 2º tan 3º .... tan 89º

= (tan 1º . tan 89º) . (tan 2º tan 88º) (tan 3º . tan 87º) ...... (tan44º tan 46º) tan 45º

$\because$ tan 89º = tan (90º – 1º) = cot 1º

= 1 . 1 . 1 . .... 1 = 1

Q.2 The value of the expression cos 1° cos 2° .... cos 179°

Ans. 0

Sol. cos1º cos2º cos179º

= cos1ºcos2º... cos 90º ... cos 179º = 0 ($\because$ cos 90º = 0)

Q.3 Find the value of tan 203° + tan 22° + tan 203° tan 22°.

Ans. 1

Sol. 203° + 22° = 225°

$\Rightarrow \quad \tan(203° + 22°) = \tan 225° = 1$

$\Rightarrow \quad \frac{\tan 203° + \tan 22°}{1 - \tan 203°.\tan 22°} = 1$

$\Rightarrow$ tan 203° + tan22° + tan 203° . tan22° = 1

Q.4 If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, find the value of xy + yz + zx.

Ans. **0**

Sol. $x = y\left(-\frac{1}{2}\right) = z\left(-\frac{1}{2}\right) \Rightarrow \frac{x}{-1} = \frac{y}{2} = \frac{z}{2} = \lambda$ (say)

$\therefore \quad xy + yz + zx = -2\lambda^2 + 4\lambda^2 - 2\lambda^2 = 0$

Q.5 In any triangle ABC, which is not right angled find the value of Σ cos A . cosec B . cosec C.

Ans. **2**

Sol. $\Delta ABC, \Sigma \cos A \cdot \operatorname{cosec} B \cdot \operatorname{cosec} C$

$$= \sum \frac{\cos A}{\sin B \sin C} \times \frac{\sin A}{\sin A} = \sum \frac{2 \sin A \cos A}{2 \sin A \sin B \sin C}$$

$$= \sum \frac{\sin 2A}{2 \sin A \sin B \sin C} = \frac{\sin 2A + \sin 2B + \sin 2C}{2 \sin A \sin B \sin C}$$

$$= \frac{4 \sin A \sin B \sin C}{2 \sin A \sin B \sin C} = 2$$

Q.6 If $A + B + C = \pi$ & $\cos A = \cos B \cdot \cos C$ then find the value of $\tan B \cdot \tan C$.

Ans. 2

Sol. Given that,

$A + B + C = \pi$ & $\cos A = \cos B \cos C$

$\Rightarrow - \cos (B + C) = \cos B \cos C$

$\Rightarrow - \cos B \cos C + \sin B \sin C = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2 \cos B \cos C$

$$\Rightarrow \frac{\sin B \sin C}{\cos B \cos C} = 2 \Rightarrow \tan B \tan C = 2$$

Q.7 If $x \in \left(\pi, \frac{3\pi}{2}\right)$ then find the value of $4 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4 \sin^4 x + \sin^2 2x}$.

Ans. 2

Sol. $x \in \left(\pi, \frac{3\pi}{2}\right)$, then

$$4 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4 \sin^4 x + \sin^2 2x}$$

$$= 4\left(\frac{1}{\sqrt{2}} \cos \frac{x}{2} + \frac{1}{\sqrt{2}} \sin \frac{x}{2}\right)^2 + \sqrt{4 \sin^4 x + 4 \sin^2 x \cos^2 x}$$

$$= \frac{4}{2} \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2 + \sqrt{4 \sin^4 x (\sin^2 x + \cos^2 x)}$$

$= 2 (1 + \sin x) + 2 |\sin x|$ $\{x \in \text{III quadrant}\}$

$= 2 + 2 \sin x - 2 \sin x = 2$

Q.8 [2] $\dfrac{\cos 20^\circ + 8 . \sin 10^\circ \sin 50^\circ \sin 70^\circ}{\sin^2 80^\circ}$

$$= \frac{\cos 20^\circ + 8 \sin 10^\circ \sin (60^\circ - 10^\circ) \sin (60^\circ + 10^\circ)}{\sin^2 80^\circ}$$

$$= \frac{\cos 20^\circ + (8 \sin 30^\circ) / 4}{\cos^2 10^\circ}$$

$$= \frac{\cos 20^\circ + 1}{\cos^2 10^\circ} = \frac{2 \cos^2 10^\circ}{\cos^2 10^\circ} = 2$$

Q.9 5

$-\sqrt{3^2 + 4^2} \le 3 \sin x + 4 \cos x \le \sqrt{3^2 + 4^2}$

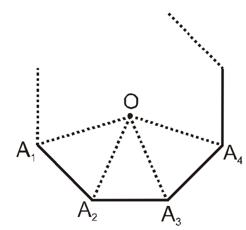
$-5 \le 3 \sin x + 4 \cos x \le 5$

Q.10 7

$$\because \quad \frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$$

$\because$ $OA_1 = OA_2 = OA_3 = OA_4 = r$ (say)

$\angle A_1OA_2 = \frac{2\pi}{n}, \angle A_1OA_3 = \frac{4\pi}{n}, \angle A_1OA_4 = \frac{6\pi}{n}$



$$\Rightarrow \frac{1}{\sin \frac{\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}} + \frac{1}{\sin \frac{3\pi}{n}}$$

$$\Rightarrow \sin \frac{2\pi}{n} \left[\sin \frac{3\pi}{n} - \sin \frac{\pi}{n}\right] = \sin \frac{3\pi}{n} . \sin \frac{\pi}{n}$$

$$\Rightarrow \sin \frac{2\pi}{n} \left[2 \cos \frac{2\pi}{n} . \sin \frac{\pi}{n}\right] = \sin \frac{3\pi}{n} \sin \frac{\pi}{n}$$

$$\Rightarrow 2 \sin \frac{2\pi}{n} \cos \frac{2\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \sin \frac{4\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \frac{4\pi}{n} = \pi - \frac{3\pi}{n}$$

$\Rightarrow 4\pi = n\pi - 3\pi \Rightarrow n = 7$

KVPY
PREVIOUS YEAR'S

Q.1 (D)

tan 81° – tan 63° – tan27° + tan9°

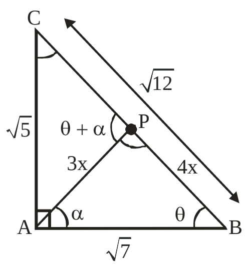
tan(90° – 9°) – tan (90° – 27°) – tan 27° + tan9°

cot 9° – cot 27° – tan 27° + tan9°

By solving we get

= 4

Q.2 (A)



$$\tan\theta = \frac{\sqrt{5}}{\sqrt{7}};\ \sin\theta = \frac{\sqrt{5}}{\sqrt{12}};\ \cos\theta = \frac{\sqrt{7}}{\sqrt{12}}$$

In ΔAPB using sine rule

$$\frac{3x}{\sin\theta} = \frac{4x}{\sin\alpha} = \frac{\sqrt{7}}{\sin(180°-(\theta+\alpha))}$$

$$\sin\alpha = \frac{4}{3}\sin\theta = \frac{4}{3}\times\frac{\sqrt{5}}{\sqrt{12}} = \frac{2\sqrt{5}}{3\sqrt{3}} = \frac{\sqrt{20}}{\sqrt{27}}$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{7}}{\sqrt{27}}$$

& $3x \sin(\theta + \alpha) = \sqrt{7}\sin\theta$

$\Rightarrow 3x(\cos\alpha + \cot\theta\ \sin\alpha) = \sqrt{7}$

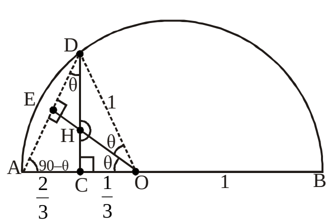
$$\Rightarrow 3x\left(\frac{\sqrt{7}}{\sqrt{27}} + \frac{\sqrt{7}}{\sqrt{5}}\frac{\sqrt{20}}{\sqrt{27}}\right) = \sqrt{7}$$

$$\Rightarrow 3x \times \frac{1}{\sqrt{3}} = 1$$

$$x = \frac{1}{\sqrt{3}}$$

$$\therefore\quad \frac{BP}{PC} = \frac{4x}{\sqrt{12}-4x} = \frac{1}{\frac{2\sqrt{3}}{4/\sqrt{3}}-1} = \frac{4}{6-4} = \frac{2}{1}$$

Q.3 (C)



'E' is mid point of AD.

$$\cos 2\theta = \frac{1}{3} \qquad \Rightarrow 2\cos^2\theta - 1 = \frac{1}{3}$$

$$\Rightarrow \cos^2\theta = \frac{2}{3} \qquad \Rightarrow \cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin\theta = \sqrt{1-\frac{2}{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{ED}{1} = \frac{1}{\sqrt{3}}$$

DH = ED sec θ

$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

Q.4 (C)

$$\frac{1}{\cos 0°\cos 1°} + \frac{1}{\cos 1°\cos 2°} + \frac{1}{\cos 2°\cos 3°} + \ldots + \frac{1}{\cos 44°\cos 45°}$$

Multiply and divided by sin1°

$$\frac{1}{\sin 1°}\left[\frac{\sin 1°}{\cos 0°\cos 1°} + \frac{\sin 1°}{\cos 1°\cos 2°} + ..\frac{\sin 1°}{\cos 44°\cos 45°}\right]$$

$$\frac{1}{\sin 1°}\left[\frac{\sin(1°-0°)}{\cos 0°\cos 1°} + \frac{\sin(2°-1°)}{\cos 1°\cos 2°} + \ldots\frac{\sin(45°-44°)}{\cos 44°\cos 45°}\right]$$

$\frac{1}{\sin 1°}$ [tan1° –tan0° +tan2° –tan1° + ...tan45° – tan44°]

$$= \frac{1}{\sin 1°}[\tan 45°]$$

$$= \frac{1}{0.0174524} = 57.2987$$

Integral part = 57

JEE MAIN
PREVIOUS YEAR'S

Q.1 (1)

$$x = 1 + \cos^2\theta + \cos^4\theta + \ldots\ldots\ldots = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$$

$$y = 1 + \sin^2\phi + \sin^4\phi + \ldots\ldots\ldots = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$$

$z = 1 + \sin^2\phi\cos^2\theta + \sin^4\phi\ \cos^4\theta + \ldots\ldots\ldots =$

$$\frac{1}{1-\sin^2\phi\cos^2\phi} = \frac{1}{1-\frac{1}{x}\frac{1}{y}} \Rightarrow z = \frac{xy}{xy-1}$$

Q.2 (4)

$$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$$

$-5 \le (k+1) \le 5$

$-6 \le k \le 4$

Q.3 **(4)**

$15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$$\tan^2\alpha = \frac{2}{3}.\ \cot^2\alpha = \frac{3}{2}$$

$\Rightarrow 27\sec^6\alpha + 8\text{cosec}^6\alpha$

$= 27(\sec^6\alpha)^3 + 8(\text{cosec}^6\alpha)^3$

$= 27(1 + \tan^2\alpha)3 + 8(1 + \cot^2\alpha)^3$

$= 250$

Q.4 (2)

Q.5 (3)

Q.6 (4)

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 0007

Let $\cot \pi / n = \theta$

$$\frac{1}{\sin\theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta} \Rightarrow \frac{\sin 3\theta - \sin\theta}{\sin\theta.\sin 3\theta} = \frac{1}{\sin 2\theta}$$

$\sin 4\theta = \sin 3\theta \Rightarrow 7\theta = \pi$

$\theta = \pi/7$, So n = 0007

Q.2 **A,C,D**

$2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$2\cos\theta + 1 = 2\sin(\theta + \phi)$

$$\frac{1}{2} = \sin(\theta+\phi) - \cos\theta$$

$\tan(2\pi - \theta) > 0$, $\tan\theta < 0$

$$\Rightarrow -1 < \sin\theta < -\frac{\sqrt{3}}{2}$$

$270^\circ < \theta < 300^\circ$

Q.3 **(BC)**

$$\cos\alpha = \left(\frac{1-a}{1+a}\right); \qquad a = \tan^2\frac{\alpha}{2}$$

$$\cos\beta = \left(\frac{1-b}{1+b}\right); \qquad b = \tan^2\frac{\beta}{2}$$

$$2\left(\left(\frac{1-b}{1+b}\right) - \left(\frac{1-b}{1+a}\right)\right) + \left(\left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)\right) = 1$$

$\Rightarrow 2((1-b)(1+a) - (1-a)(1+a)) + (1-a)(1-b) = (1+a)(1+b)$

$2(1 + a - b - ab - (1 + b - a - ab)) + 1 - a - b + ab = 1 + a + b + ab$

$4(a - b) = 2(a + b)$

$2a - 2b = a + b$

$a = 3b$

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$$

Q.4 (A,B,D)

$$f(n) = \frac{\sum_{k=0}^{n}\left(\cos\left(\frac{\pi}{n+2}\right) - \cos\left(\frac{2k+3}{n+3}\right)\pi\right)}{\sum_{k=0}^{n}\left(1-\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

$$f(n) = \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \left(\sum_{k=0}^{n}\cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{(n+1) - \left(\sum_{k=0}^{n}\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

f(n)

$$= \frac{(n+1)\cos\frac{\pi}{n+2} - \left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left(\frac{n+3}{n+3}\right)\pi\right)}{(n+1) - \left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left(\frac{2(n+2)\pi}{2(n+2)}\right)\right)}$$

$$f(n) = \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{(n+1)+1}$$

$$\Rightarrow g(x) = \cos\left(\frac{\pi}{n+2}\right)$$

(A) $\sin\left(7\cos^{-1}\cos\frac{\pi}{7}\right) = \sin\pi = 0$

(B) $f(4) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(C) $\lim\limits_{x\to\infty}\cos\left(\frac{\pi}{n+2}\right) = 1$

(D) $\alpha = \tan\left(\cos^{-1}\cos\frac{\pi}{8}\right) = \sqrt{2} - 1 \Rightarrow \alpha + 1 = \sqrt{2}$

$\alpha^2 + 2\alpha - 1 = 0$

Trigonometric Equation

EXERCISES

ELEMENTARY

Q.1 (3)

$2\tan^2\theta = \sec^2\theta \Rightarrow 2\tan^2\theta = \tan^2\theta + 1$

$\Rightarrow \tan^2\theta = 1 = \tan^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

Q.2 (2)

$\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}$

Q.3 (1)

$\tan 2\theta = \cot\theta \Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}$

Q.4 (2)

$4 + 2\sin^2 x = 5$

$\Rightarrow \sin^2 x = \frac{1}{2} = \sin^2\frac{\pi}{4} \Rightarrow x = n\pi \pm \frac{\pi}{4}$

Q.5 (1)

We have $\frac{\pi}{4}\cot\theta = \frac{\pi}{2} - \frac{\pi}{4}\tan\theta \Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow \sin 2\theta = 1 = \sin\frac{\pi}{2} \Rightarrow \theta = n\pi + \frac{\pi}{4}$

Q.6 (4)

$\cos 2\theta = \cos\left(\frac{\pi}{2} - \alpha\right) \Rightarrow 2\theta = 2n\pi \pm \left(\frac{\pi}{2} - \alpha\right)$

$\Rightarrow \theta = n\pi \pm \left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

Q.7 (1)

$\tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\Rightarrow 7\theta = n\pi + \frac{\pi}{2} \Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$

Q.8 (2)

$(2\cos x - 1)\,(3 + 2\cos x) = 0$

Then, $\cos x = \frac{1}{2}$ as $\cos x \neq \frac{-3}{2}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3};\ \begin{Bmatrix} \text{for } n = 0,\ x = \frac{\pi}{3}, \frac{5\pi}{3} \\ \text{for } n = 1,\ x = \frac{5\pi}{3} \end{Bmatrix}$

Q.9 (3)

$\cot\theta + \tan\theta = 2\ \text{cosec}\theta \Rightarrow \frac{2}{\sin\theta} = \frac{1}{\sin\theta\cos\theta}$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\sin\theta = 0 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi$

Q.10 (3)

$\sec^2\theta + \tan^2\theta = \frac{5}{3}$, also $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3} = \tan^2\left(\frac{\pi}{6}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

Q.11 (1)

$\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$

$\Rightarrow \tan\theta(\tan\theta - 1) - \sqrt{3}(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - \sqrt{3})\,(\tan\theta - 1) = 0 \Rightarrow \theta = n\pi + \frac{\pi}{3},\ n\pi + \frac{\pi}{4}$

Q.12 (3)

$2\sqrt{3}\cos^2\theta = \sin\theta \Rightarrow 2\sqrt{3}\sin^2\theta + \sin\theta - 2\sqrt{3} = 0$

$\Rightarrow \sin\theta = \frac{-1 \pm 7}{4\sqrt{3}} \Rightarrow \sin\theta = \frac{-8}{4\sqrt{3}}$, (Impossible)

and $\sin\theta = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{3}$

Q.13 (1)

$4 - 4\cos^2\theta + 2(\sqrt{3} + 1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4\cos^2\theta - 2(\sqrt{3} + 1)\cos\theta + \sqrt{3} = 0$

$\Rightarrow \cos\theta = \frac{2(\sqrt{3} + 1) \pm \sqrt{4(\sqrt{3} + 1)^2 - 16\sqrt{3}}}{8}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $1/2 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$ or $2n\pi \pm \pi/3$

Q.14 (1)

$2 - 2\cos^2\theta + \sqrt{3}\cos\theta + 1 = 0$

$\Rightarrow 2\cos^2\theta - \sqrt{3}\cos\theta - 3 = 0$

$\Rightarrow \cos\theta = \frac{\sqrt{3} \pm \sqrt{3 + 24}}{4} = \frac{\sqrt{3}(1 \pm 3)}{4} = \sqrt{3}\left(-\frac{1}{2}\right)$

$\Rightarrow \theta = \frac{5\pi}{6}$

Q.15 (2)

$3\sin^2 x + 10\cos x - 6 = 0$

$3(1-\cos^2 x) + 10\cos x - 6 = 0$

On solving, $(\cos x - 3)(3\cos x - 1) = 0$

Either $\cos x = 3$, (which is not possible)

or $\cos x = \frac{1}{3}$

Q.16 (4)

We have, $\cos^2\theta + \sin\theta + 1 = 0$

$\Rightarrow 1 - \sin^2\theta + \sin\theta + 1 = 0$

$\Rightarrow \sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = 2$, which is not possible and $\sin\theta = -1$.
Therefore, solution of given equation lies in the

interval $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$.

Q.17 (3) $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$12(\operatorname{cosec}^2\theta - 1) - 31\operatorname{cosec}\theta + 32 = 0$

$12\operatorname{cosec}^2\theta - 31\operatorname{cosec}\theta + 20 = 0$

$12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$(4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3}$; $\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}$

Q.18 (2)

$5 - 5\sin^2\theta + 7\sin^2\theta = 6 \Rightarrow 2\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

Q.19 (3)

$\sin 4\theta = \cos\theta - \cos 7\theta \Rightarrow \sin 4\theta = 2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = n\pi$ or $\sin 3\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 3\theta = n\pi + (-1)^n \frac{\pi}{6} \Rightarrow \theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$

Q.20 (4)

$\cos\theta + \cos 2\theta + \cos 3\theta = 0$

$\Rightarrow (\cos\theta + \cos 3\theta) + \cos 2\theta = 0$

$\Rightarrow 2\cos 2\theta\cos\theta + \cos 2\theta = 0 \Rightarrow$

$\cos 2\theta(2\cos\theta + 1) = 0$

$\Rightarrow \cos 2\theta = 0 = \cos\frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} \Rightarrow \theta = 2m\pi \pm \frac{\pi}{4}$

or $\cos\theta = \frac{-1}{2} = \cos\frac{2\pi}{3} \Rightarrow \theta = 2m\pi \pm \frac{2\pi}{3}$.

Q.21 (2)

$\tan(\cot x) = \cot(\tan x)$

$\Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$

$\Rightarrow \frac{2}{\sin 2x} = n\pi + \frac{\pi}{2} \Rightarrow \sin 2x = \frac{2}{n\pi + \frac{\pi}{2}} = \frac{4}{(2n+1)\pi}$

Q.22 (4)

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2}$ {dividing by

$\sqrt{(\sqrt{3})^2 + 1^2} = 2$}

$\Rightarrow \sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}$

Q.23 (4)

$3\cos x + 4\sin x = 6$

$\Rightarrow \frac{3}{5}\cos x + \frac{4}{5}\sin x = \frac{6}{5} \Rightarrow \cos(x - \theta) = \frac{6}{5}$

[where $\theta = \cos^{-1}(3/5)$]

So, that equation has no solution.

Q.24 (4)

Given equation is $\sqrt{3}\sin x + \cos x = 4$

which is of the form $a\sin x + b\cos x = c$ with

$a = \sqrt{3}, b = 1, c = 4$.

Here $a^2 + b^2 = 3 + 1 = 4 < c^2$, therefore the given equation has no solution.

Q.25 (1)

$f(x) = \cos x - x + \frac{1}{2}$, $f(0) = \frac{3}{2} > 0$

$f\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} + \frac{1}{2} = \frac{1-\pi}{2} < 0 \quad \left(\because \pi = \frac{22}{7} \text{ nearly}\right)$

$\therefore$ One root lies in the interval $\left[0, \frac{\pi}{2}\right]$.

Q.26 (3)

$\sec x \cos 5x = -1 \Rightarrow \cos 5x = -\cos x$

$\Rightarrow 5x = 2n\pi \pm (\pi - x) \Rightarrow x = \frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{4}$

Hence, $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6}, \frac{11\pi}{6}$

Q.27 (4)

$$\sin\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) \Rightarrow \theta = \left(\pi + \frac{\pi}{6}\right)$$

Hence general value of θ is $2n\pi + \frac{7\pi}{6}$

Q.28 (2)

$$\cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}, \frac{5\pi}{4} \; ; \; \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

∴ The general value is $2n\pi + \frac{5\pi}{4}$ or $(2n+1)\pi + \frac{\pi}{4}$

Q.29 (2)

$$\tan\theta = \frac{-1}{\sqrt{3}} = \tan\left(\pi - \frac{\pi}{6}\right), \; \sin\theta = \frac{1}{2} = \sin\left(\pi - \frac{\pi}{6}\right)$$

and $\cos\theta = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$

Hence principal value is $\theta = \frac{5\pi}{6}$

Q.30 (3)

$$1 + \cos^2\theta + \cos^4\theta + ... \infty = 4 \Rightarrow \frac{1}{1-\cos^2\theta}$$

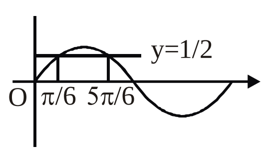
$= 4$ ∴ $\text{cosec}^2\theta = 4 \Rightarrow \sin^2\theta = \frac{1}{4} = \sin^2\frac{\pi}{6}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

Q.31 (3)

$\sin x \le \frac{1}{2}$

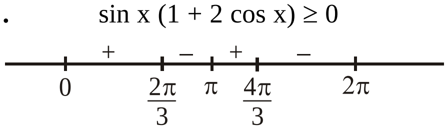
$$\Rightarrow x \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$$



Possible integral values of x are 0, 3, 4, 5, 6.

Q.32 (A)

. $\sin x(1 + 2\cos x) \ge 0$



JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

$4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$

$\Rightarrow 2\cos\theta(2\sin\theta - 1) - \sqrt{3}(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(2\cos\theta - \sqrt{3}) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

Q.2 $2\sin\theta + \tan\theta = 0 \Rightarrow \sin\theta = 0$ or $2 + \frac{1}{\cos\theta} = 0$

$\Rightarrow \theta = n\pi$ or $2 = -\frac{1}{\cos\theta} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 2m\pi$

$\pm \frac{2\pi}{3}$.

Q.3 (1)

Given $\cot 3\theta - \cot\theta = 0$

$\Rightarrow \cot 3\theta = \cot\theta$

$\Rightarrow 3\theta = n\pi + \theta \; ; n \in I$

$\Rightarrow 2\theta = n\pi \; ; n \in I$

$$\Rightarrow \theta = \frac{n\pi}{2} \because n \in I \begin{cases} 3\theta \ne n\pi \text{ or } \theta \ne m\pi \\ \theta \ne \frac{n\pi}{3} \end{cases}$$

$\Rightarrow \theta = (2n-1) . \frac{\pi}{2} \; ; n \in I$

Q.4 (1)

$2\cos^2(\pi + x) + 3\sin(\pi + x) = 0$

$\Rightarrow 2\cos^2 x - 3\sin x = 0 \Rightarrow 2 - 2\sin^2 x - 3\sin x = 0$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow \sin x = -2, \frac{1}{2}$

$\Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

Q.5 (1)

$\cos 2\theta + 3\cos\theta = 0 \Rightarrow 2\cos^2\theta + 3\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = \frac{3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$

As $-1 \le \cos\theta \le 1$ a ∴ $\cos\theta = \frac{-3+\sqrt{17}}{4}$ only

$\Rightarrow \theta = 2n\pi \pm \alpha$ where $\cos\alpha = \frac{\sqrt{17}-3}{4}$

Q.6 (2)

$\sin\theta + 7\cos\theta = 5$

$\Rightarrow \frac{2t}{1+t^2} + \frac{7(1-t^2)}{1+t^2} = 5$ where $t = \tan\left(\frac{\theta}{2}\right)$

$\Rightarrow 2t + 7 - 7t^2 = 5 + 5t^2$

$\Rightarrow \tan\frac{\theta}{2}$ is root of $12t^2 - 2t - 2 = 0$

or $6t^2 - t - 1 = 0$.

Q.7 (4)

Given

$\sin 3\theta = 4\sin\theta \cdot \sin 2\theta \sin 4\theta$, where $0 \le \theta \le \pi$

$\Rightarrow 3\sin\theta - 4\sin^3\theta - 4\sin\theta \sin 2\theta \sin 4\theta = 0$

$\Rightarrow \sin\theta\,[3 - 4\sin^2\theta - 4\sin 2\theta \sin 4\theta]$

$\Rightarrow \sin\theta\,[3 - 2 + 2\cos 2\theta - 2\cos 2\theta + 2\cos 6\theta]$

$\Rightarrow \sin\theta\,[1 + 2\cos 6\theta] = 0$

$\Rightarrow \sin\theta = 0$ or $\cos 6\theta = -\frac{1}{2}$

$\theta = 0, \pi$ or

$0 \le \theta \le \pi$

$0, \pi \in [0, \pi]$ or $0 \le 6\theta \le 6\pi$ (3 rounds)

Number of solutions in 1 round = 2

total number of solutions in $\theta \in [0, \pi]$

= 2 + 6 = 8

Q.8 (3)

$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$

$\Rightarrow 3\tan 3x = 3 \Rightarrow \tan 3x = 1$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

Q.9 (2)

Given, $2\cos 2x = 3.2\cos^2 x - 4$

$\Rightarrow 2\cos 2x = 3(\cos 2x + 1) - 4$

$\Rightarrow \cos 2x = 1 \Rightarrow 2x = 2n\pi$; $n \in I$

$\Rightarrow x = n\pi$; $n \in I$

Q.10 (3)

$\sin x + \sin 5x = \sin 2x + \sin 4x$

$\Rightarrow 2\sin 3x \cos 2x = 2\sin 3x \cos x$

$\Rightarrow \sin 3x = 0$ or $\cos 2x = \cos x$

$\Rightarrow 3x = n\pi$ or $2x = 2n\pi \pm x$

$\Rightarrow x = \frac{n\pi}{3}, \; 2n\pi, \; \frac{2n\pi}{3}$

$\Rightarrow x = \frac{n\pi}{3}$ (It includes all three possible)

Q.11 (2)

$\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2} \Rightarrow 2(4\cos^3\theta - 3\cos\theta) = 2(2\cos^2\theta - 1) - 1$

$\Rightarrow 8\cos^3\theta - 4\cos^2\theta - 6\cos\theta + 3 = 0$

$\Rightarrow (4\cos^2\theta - 3)(2\cos\theta - 1) = 0 \Rightarrow \cos\theta = \frac{1}{2}, \pm\frac{\sqrt{3}}{2}$

But when $\cos\theta = \pm\frac{\sqrt{3}}{2}$ then $2\cos 2\theta - 1 = 0$

$\therefore$ rejecting this value, $\cos\theta = \frac{1}{2}$ is valid only

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

Q.12 (1)

Given $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \times \tan 2x} = 1$

$\Rightarrow \tan(3x - 2x) = 1 \Rightarrow \tan x = 1 = \tan\frac{\pi}{4}$

$\Rightarrow x = n\pi + \frac{\pi}{4}\; n \in I$

But at this value of x, $\tan 2x = \infty$

which is not acceptable $\therefore x \in \phi$

Q.13 (3)

$3\sin\theta - 4\cos\theta = 4a - 3$

on divided by 5, we get

$\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta = \frac{4a-3}{5}$

$\Rightarrow \sin(\theta - \alpha) = \frac{4a-3}{5}$ ($\tan\alpha = \frac{4}{3}$)

Equation has solution, then

$\Rightarrow -1 \le \frac{4a-3}{5} \le 1$ $\Rightarrow -5 \le 4a - 3 \le 5$

$\Rightarrow -2 \le 4a \le 8$ $\Rightarrow \frac{-1}{2} \le a \le 2$

Positive integral value 1, 2.

Q.14 (4)

Given $\sin\theta + 2\sin 2\theta + 3\sin\theta + 4\sin 4\theta = 10$ in $(0, \pi)$

Using boundary of SM

It's only possible if $\Rightarrow 1 + 2 + 3 + 4 = 10$

(Using boundary values)

$\Rightarrow \sin\theta = 1$ & $\sin 2\theta = 1$ & $\sin 3\theta = 1$ & $\sin 4\theta = 1$

$\theta = 2m\pi + \frac{\pi}{2}, 2\theta = 2n\pi + \frac{\pi}{2}, 3\theta = 2k\pi + \frac{\pi}{2},$

$4\theta = 2p\pi + \frac{\pi}{2}$

$m \in I \; n \in I \; k \in I \; p \in I$

$\theta = \frac{\pi}{2} \; \theta = \frac{\pi}{4} \; \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{\pi}{8}, \frac{5\pi}{8}$

$\therefore \quad x \in \left\{\frac{\pi}{2}\right\} \cap \left\{\frac{\pi}{4}\right\} \cap \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \cap \left\{\frac{\pi}{8}, \frac{5\pi}{8}\right\}$

$x \in \{\;\}$ number of solutoins is zero.

Q.15 (3)

Given, $\tan\theta = -1$ & $\cos\theta = \frac{1}{\sqrt{2}}$

principal solution

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ & $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

common principal solutoin is $\frac{7\pi}{4}$

then general solution is $\theta = 2n\pi + \frac{7\pi}{4}$, $n \in I$

Q.16 (1)

$\because \cot^3\theta + 3\sqrt{3} = 0 \Rightarrow \cot\theta = -\sqrt{3} \Rightarrow \tan\theta = \frac{-1}{\sqrt{3}}$

and $\text{cosec}^5\theta + 32 = 0 \Rightarrow \text{cosec}\,\theta = -2 \Rightarrow \sin\theta = \frac{-1}{2}$

$\therefore$ Common value of $\theta = \frac{-\pi}{6}$ in $(-\pi, \pi]$

$\therefore$ General solution will be $\theta = 2n\pi - \frac{\pi}{6}$, $n \in I$.

Q.17 (2)

Given $4\cos^3x - 4\cos^2x - \cos(\pi + x) - 1 = 0$

$\Rightarrow 4\cos^3x - 4\cos^2x + \cos x - 1 = 0$

A.M. of roots $x \in [0, 315]$

$\Rightarrow \; 4\cos^2x(\cos x - 1) + (\cos x - 1) = 0$

$\Rightarrow \; (\cos x - 1)(4\cos^2x + 1) = 0$

$\Rightarrow \; \cos x = 1$ or $\cos^2 x = -\frac{1}{4}$ (not possible)

$x = 2n\pi, n \in I$

$0 \le 2n \le 100 x \in [0, 315]$

$0 \le n \le 50$ or $[0, 100\pi]$

$x = 0, 2\pi, 4\pi, 6\pi, \ldots, 100\pi$

$\text{A.M.} = \frac{2\pi[0+1+2+3+\ldots+50]}{51}$

$= \frac{2\pi}{51} \; \frac{50\times 51}{2} = 50\pi$

Q.18 (4)

Given $a_1 + a_2\cos 2x + a_3\sin^2x = 0$

$a_1 + a_2(1 - 2\sin^2x) + a_3\sin^2x = 0$

$(a_1 + a_2) = (2a_2 - a_3)\sin^2x$

$\sin^2x = \frac{a_1 + a_2}{2a_2 - a_3} \quad \because \quad 0 \le \sin^2x \le 1$

$0 \le \frac{a_1 + a_2}{2a_2 - a_3} \le 1$

$0 \le a_1 + a_2 \le 2a_2 - a_3$

$a_1 + a_2 \ge 0$

....(i)

$2a_2 - a_3 \ge 0$

....(ii)

$-a_1 + a_2 - a_3 \ge 0$

...(iii)

$N^r \le D^r$

homogenous system of equations

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$|A| = 1(-2 + 1) - 1(0 - 1) + 0 = -1 + 1 = 0$

So number of solution is infinite

Q.19 (4)

Given $|\cos x| = \cos x - 2\sin x$

Case-I : if $\cos x \ge 0$, then $\cos x = \cos x - 2\sin x$

$\Rightarrow \sin x = 0$

$\Rightarrow x = n\pi$ but $\cos x \ge 0$

so only even integral multiple of π is acceptable

$\therefore \; x = 2n\pi \;;\; n \in I$

Case-II :

if $\cos x < 0$, then $-\cos x = \cos x - 2\sin x - 2\cos x = -2\sin x$

$\Rightarrow \; \tan x = 1$

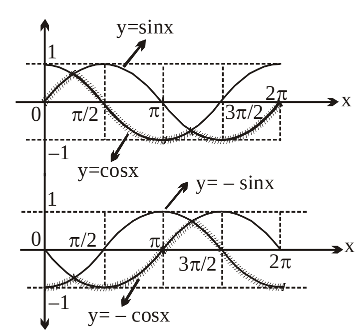
$\Rightarrow \; x = n\pi + \frac{\pi}{4}$ (x can be in I & III quadrant)

But $\cos x < 0$ So x will be in III quadrant

s.t. n should be odd integer

$x = (2m + 1)\pi + \frac{\pi}{4}$, $m \in I$

Q.20 (2)



Clearly from the graph $x \in \left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

Ans.]

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OBJECTIVE QUESTIONS

Q.1 (B)

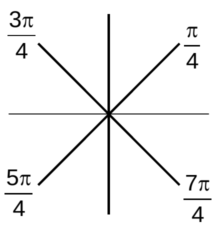
Given $\sin x \sqrt{8\cos^2 x} = 1$

$\Rightarrow 8\sin^2 x \cos^2 x = 1 \quad \Rightarrow 2\sin^2 2x = 1$

$\Rightarrow \sin^2 2x = \frac{1}{2} \quad x \in [0, 2\pi]$

$\Rightarrow \sin 2x = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow 2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

Common difference

$d = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{\pi}{4}$

Q.2 (A)

Given $2\cos x = \sqrt{2+2\sin 2x}$; $x \in [0, 2\pi)$

$\Rightarrow 4\cos^2 x = 2 + 2\sin 2x \Rightarrow 2\cos^2 x - 1 = \sin 2x$

$\Rightarrow \cos 2x = \sin 2x \Rightarrow \cos 2x = \cos\left(\frac{\pi}{2} - 2x\right)$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$

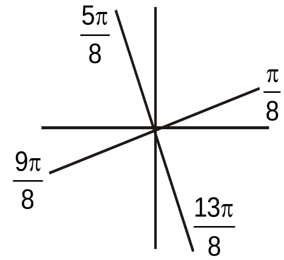
$\Rightarrow 2x = 2n\pi + \frac{\pi}{2} - 2x$ & $2x = 2n\pi - \frac{\pi}{2} + 2x$

(not possible)

$\therefore \quad 4x = \left(\frac{4n+1}{2}\right)\pi \Rightarrow x = \left(\frac{4n+1}{8}\right)\pi$

$\therefore 0 \le \left(\frac{4n+1}{8}\right)\pi < 2\pi \Rightarrow 0 \le \frac{4n+1}{8} < 2$

$\Rightarrow -\frac{1}{8} \le \frac{n}{2} < 2 - \frac{1}{8} \Rightarrow -\frac{1}{4} \le n < \frac{15}{4}$



n = 0, 1, 2, 3

$x \in [0, 2\pi)$

$x = \frac{\pi}{4}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

For $x = \frac{5\pi}{8}$ and $x = \frac{9\pi}{8}$, cosx will give negative value

$x = \frac{\pi}{8}, \frac{13\pi}{8}$

Q.3 (C)

$\tan^2\alpha + 2\sqrt{3}\tan\alpha - 1 = 0$

$\Rightarrow \tan\alpha = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{-2\sqrt{3} \pm 4}{2} = -\sqrt{3}$

± 2

$= 2 - \sqrt{3}, -\left(2 + \sqrt{3}\right)$

$= \tan 15^\circ, -\cot 15^\circ$

$= \tan\frac{\pi}{12}, \tan\left(\frac{-5\pi}{12}\right)$

$\Rightarrow \alpha = n\pi + \frac{\pi}{12}, n\pi - \frac{5\pi}{12}$

or $n\pi + \frac{\pi}{12}, (2n-1)\frac{\pi}{2} + \frac{\pi}{12}$

$\Rightarrow \alpha = \frac{2n\pi}{2} + \frac{\pi}{12}, (2n-1)\frac{\pi}{2} + \frac{\pi}{12}$

$\Rightarrow \alpha = \frac{n\pi}{2} + \frac{\pi}{12}$.

Q.4 (D)

Given $20\sin^2\theta + 21\cos\theta - 24 = 0$ & $\frac{7\pi}{4} < \theta < 2\pi$

$\Rightarrow 20 - 20\cos^2\theta + 21\cos\theta - 24 = 0$

$\Rightarrow 20\cos^2\theta - 21\cos\theta + 4 = 0$

$\Rightarrow (5\cos\theta - 4)(4\cos\theta - 1) = 0$

$\Rightarrow \cos\theta \ne \frac{1}{4} \quad \because \theta \notin \left(\frac{7\pi}{4}, 2\pi\right)$

$\cos\theta = \frac{4}{5}$ Now $\quad \cos\frac{\theta}{2} = \sqrt{\frac{\cos\theta + 1}{2}}$

$\cos\frac{\theta}{2} = \pm\frac{3}{\sqrt{10}}$

$\cos\frac{\theta}{2} = -\frac{3}{\sqrt{10}} \quad \because \frac{\theta}{2} \in \left(\frac{7\pi}{8}, \pi\right)$

$\sin\frac{\theta}{2} = \frac{1}{\sqrt{10}}$

$\Rightarrow \cot\frac{\theta}{2} = -3$

Q.5 (B)

Given, $\sin(2A + B) = \frac{1}{2} \Rightarrow \sin(2A + B) = \sin\frac{\pi}{6}$

or $\sin\frac{5\pi}{6}$

But $2A + B = \frac{\pi}{6}$ is not possible

$\therefore \quad 2A + B = \frac{5\pi}{6}$ ....(i)

Given that
$2B = A + C$
....(ii)
We know
$A + B + C = \pi$ .... (iii)

From (ii) & (iii) $3B = \pi \Rightarrow B = \frac{\pi}{3}$

From (i) $2A = \frac{5\pi}{6} - \frac{\pi}{3} \Rightarrow A = \frac{\pi}{4}$

From (iii) $C = \pi - (A + B) = \pi - \left(\frac{\pi}{3} + \frac{\pi}{4}\right) \Rightarrow C = \frac{5\pi}{12}$

Q.6 (C)

$\sin x(\sin x + \cos x) = \frac{n}{4} \Rightarrow 2\sin^2 x + 2\sin x\cos x = \frac{n}{2}$

$1 - \cos 2x + \sin 2x = \frac{n}{2} \Rightarrow \sin 2x - \cos 2x = \frac{n}{2} - 1$

For the solution to be exists

$-\sqrt{2} \le \frac{n}{2} - 1 \le \sqrt{2}$

$\Rightarrow 2(1 - \sqrt{2}) \le n \le 2(\sqrt{2} + 1)$

$\Rightarrow -0.828 \le n \le 4.828$

$\therefore$ Number of integral values of n is 5

Q.7 (D)

$2\cos^2 2x - 2\cos 2x\cos\left(\frac{2014\pi^2}{x}\right) = -2\sin^2 2x$

$\Rightarrow 2 = 2\cos 2x\cos\left(\frac{2014\pi^2}{x}\right) = 1$

Hence, $\cos 2x = 1$ and $\cos\left(\frac{2014\pi^2}{x}\right) = 1$.

Both equals to – 1 reject.

$\therefore x = n\pi$ and $\frac{2014\pi^2}{x} = 2k\pi, k \in I$

$x = \frac{1007\pi}{k} \Rightarrow 1007 = 19 \times 53 \times 1$

$\therefore x = \pi, 53\pi, 19\pi, 1007\pi$

Sum $= \pi(1 + 19 + 53 + 1007) = 1080\pi$.

Q.8 (C)

Given, $2\sqrt{\cos^6\theta - \sin^6\theta} = (1 + \cos 2\theta)$ ....(1)

Now squaring on both sides, we get

$4\left(1 - \frac{3}{4}\sin^2 2\theta\right) = (1 + 2\cos 2\theta + \cos^2 2\theta)$

$\Rightarrow 4 - 3(1 - \cos^2 2\theta) = 1 + 2\cos 2\theta + \cos^2 2\theta \Rightarrow 2\cos 2\theta(\cos 2\theta - 1) = 0$

$\therefore$ Either $\cos 2\theta = 0$ or $\cos 2\theta = 1$

Now, $\cos 2\theta = 1 \Rightarrow 2\theta = 2m\pi, m \in I \Rightarrow \theta = m\pi, m \in I$

Also, $\cos 2\theta = 0 \Rightarrow 2\theta = (2n + 1)\frac{\pi}{2}, n \in I \Rightarrow \theta = (2n + 1)\frac{\pi}{4}, n \in I$

But, for $\theta = (2n + 1)\frac{\pi}{4}, n \in I$

L.H.S. of equation (1) equals 0 and R.H.S. of equation (2) equals 1

So, L.H.S. $\neq$ R.H.S.
Hence, $\theta = m\pi$, $m \in I$
So, possible solutions are $\theta = -\pi, 0, \pi$.
Hence, number of solution are three.

Q.9 (C)

$$\cos^2\left(\frac{\pi}{4}(\cos x + \sin x)\right)$$

$$= 1 + \tan^2\left(x + \frac{\pi}{4}\tan^2 x\right)$$

$$\Rightarrow \cos^2\left(\frac{\pi}{4}(\cos x + \sin x)\right) = 1 \text{ and}$$

$$\tan^2\left(x + \frac{\pi}{4}\tan^2 x\right) = 0 \qquad \text{.......(1)}$$

$$\Rightarrow \frac{\pi}{4}(\cos x + \sin x) = n\pi$$

$\Rightarrow \cos x + \sin x = 4n \Rightarrow \cos x + \sin x = 0$

$$\Rightarrow \tan x = -1 \Rightarrow x = n\pi - \frac{\pi}{4}$$

all of which satisfy (1) also
$\therefore$ Number of solutions in $[-2\pi, 2\pi]$ is 4.

Q.10 (C)

$$\underbrace{2\sin^2\alpha + 2^{-\cos^2\beta}}_{\le 3} = \underbrace{e^{\gamma} + \frac{1}{e^{\gamma}} + 1}_{\ge 3}$$

The equation will satisfy when $\sin^2\alpha = 1$, $\cos^2\beta = 0$ and $e^{\gamma} = 1$

$$\Rightarrow \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \ \beta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ and } \gamma = 0$$

$\therefore$ Number of ordered triplets (α, β, γ) is 4. **Ans.**]

Q.11 (D)

$\cos 2x + a \sin x = 2a - 7$
$\Rightarrow 1 - 2\sin^2 x + a\sin x = 2a - 7$
$\Rightarrow 2\sin^2 x - a\sin x + 2(a - 4) = 0$ .....(1)

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 16(a-4)}}{4}$$

$$= \frac{a \pm \sqrt{(a-8)^2}}{4} = \frac{a \pm (a-8)}{4} = \frac{2a-8}{4}, \frac{8}{4}$$

$$\Rightarrow \sin x = \frac{a-4}{2} \Rightarrow -1 \le \frac{a-4}{2} \le 1$$

$\Rightarrow 2 \le a \le 6$
$\Rightarrow a = 2,3,4,5,6$
$\therefore$ No. of solutions = 5

Q.12 (B)

Given $4\,\text{cosec}^2(\pi(a + x)) + a^2 - 4a = 0$
$\Rightarrow 4\,\text{cosec}^2(\pi(a + x)) = 4a - a^2$

$$\Rightarrow \sin^2(\pi(a + x)) = \frac{4}{4a - a^2}$$

$0 \le \sin^2 x \le 1$

$$0 \le \frac{4}{4a - a^2} \le 1$$

$0 \le 4 \le 4a - a^2$
$\therefore 4a - a^2 \ge 4$
$\Rightarrow (a-2)^2 \ge 0$
$a \ge 2$
if $a = 1$
$4 \le (4 - 1)1$
$4 \not\le 3$
$\Rightarrow$ $a < 2$ does not satisfy the inequation
$\therefore$ $a = 2$

Q.13 (C)

Period of $\sin 2x = \pi$
and period of $\tan x = \pi$

$\therefore$ Common period = LCM of $(\pi, \pi) = \pi$

$$\therefore 2\sin 2x + \sqrt{3} = 0 \Rightarrow \sin 2x = \frac{-\sqrt{3}}{2} \Rightarrow 2x = \frac{4\pi}{3}, \frac{5\pi}{3} \Rightarrow x = \frac{2\pi}{3}, \frac{5\pi}{6}$$

$$\text{and } \sqrt{3}\tan x + 1 = 0 \Rightarrow \tan x = \frac{-1}{\sqrt{3}} \Rightarrow x = \frac{5\pi}{6}$$

$\therefore$ The most general value of x given by x = (common period) n + common value

$$= n\pi + \frac{5\pi}{6}, n \in I.$$

Q.14 (C)

$$\cos^2 \pi x - \sin^2 \pi y = \frac{1}{2}$$

$$\cos \pi(x + y) \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$\cos(\pi)(x + y) = 1$
$\Rightarrow \pi(x + y) = 2n\pi$; where $n \in I$

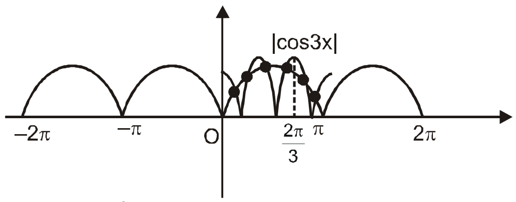
$$\Rightarrow x + y = 2n \text{ and } x - y = \frac{1}{3}$$

$$\Rightarrow x = n + \frac{1}{6};\ y = n - \frac{1}{6};\ (n \in I)$$

$(x, y) \in \left(n+\frac{1}{6}, n-\frac{1}{6}\right)$ which is satisfied by 3rd option for n = 2.]

Q.15 (C)

|sinx| = | cos3x|



Number of solutions in [0, π] = 6

∴ Number of solutions in [–2π, 2π] = 24

Q.16 (B)

$a - a\cos^2 x + |\cos x| - 2a = 0$

$\Rightarrow a = \frac{|\cos x|}{1+\cos^2 x}$

(For a = 0 equation has solution)

$= \frac{1}{|\cos x| + \left|\frac{1}{\cos x}\right|}$

$|\cos x| \in [0, 1] \Rightarrow |\cos x| + \frac{1}{|\cos x|} \in [2, 0)$

$\Rightarrow a \in \left(0, \frac{1}{2}\right]$

Hence complete set of values of a is $\left[0, \frac{1}{2}\right]$

Q.17 (C)

$(\sin 3x + \cos 2y)^2 + (\cos 2y + \tan 4z)^2 + (\tan 4z + \sin 3x)^2 \le 0$

sin 3x = 0, cos 2y = 0, tan 4z = 0

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \quad y = \frac{\pi}{4}, \frac{3\pi}{4}, \quad z = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

Therefore total number of ordered triplets (x, y, z) = 24

Q.18 (C)

$2^{\left(\sqrt{\sin^2 x - 2\sin x + 5} - 2\sin^2 y\right)} \le 1$

$\sqrt{\sin^2 x - 2\sin x + 5} - 2\sin^2 y \le 0$

$\underbrace{\sqrt{(\sin x - 1)^2 + 4}}_{\ge 2} \le \underbrace{2\sin^2 y}_{\le 2}$

$\therefore \sin x = 1 \Rightarrow x = \frac{\pi}{2}$ & $\sin^2 y = 1$

$\Rightarrow \sin y = \pm 1 \quad \Rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}$

Hence number of ordered pair (x, y) in [0, 2π] is 2.

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MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B, D)

$\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$

$\Rightarrow x - y = \frac{\pi}{6}, \frac{5\pi}{6}$ and $x + y = \frac{\pi}{3}, \frac{5\pi}{3}$

Adding $2x = \frac{\pi}{2}$ or $\frac{7\pi}{6}$ or $\frac{11\pi}{6} \Rightarrow x = \frac{\pi}{4}$ or $\frac{7\pi}{12}$ or $\frac{11\pi}{12}$

when $x = \frac{\pi}{4}, y = \frac{\pi}{12}$

when $x = \frac{7\pi}{12}$ no value of y is possible.

whev $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$.

Q.2 (A, B, C, D)

cos 15x = sin 5x

coσ 15x = cos $\left(\frac{\pi}{2} - 5x\right)$ or cos $\left(\frac{3\pi}{2} + 5x\right)$

$15x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right)$ or $15x = 2n\pi \pm \left(\frac{3\pi}{2} + 5x\right)$

$\Rightarrow x = \frac{n\pi}{10} + \frac{\pi}{40}, n \in I, x = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in I$

and $x = \frac{n\pi}{5} - \frac{\pi}{20}, n \in I$ and $x = \frac{n\pi}{10} - \frac{3\pi}{40}, n \in I$

Q.3 (B, C)

$\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow \sin^2 x - (1 - 2\sin^2 x) = 2 - 2\sin x \cos x$

$\Rightarrow 3\sin^2 x + 2\sin x \cos x = 3$

case-I : $\cos x \ne 0 \therefore 3\tan^2 x + 2\tan x = 3(1 + \tan^2 x)$

$\Rightarrow \tan x = \frac{3}{2}$

case-II : cos x = 0 ∴ 3(1) + 2(±1) (0) = 3 which is true

$\therefore x = (2n + 1)\frac{\pi}{2}$

Q.4 (C, D)

$\sin^2 x + 2 \sin x \cos x - 3 \cos^2 x = 0$

case-I : $\cos x \neq 0$

$\therefore \tan^2 x + 2 \tan x - 3 = 0$

$\Rightarrow \tan x = 3, 1 \Rightarrow x = n\pi + \tan^{-1}(-3), n\pi + \frac{\pi}{4}$

case-II : $\cos x = 0 \Rightarrow 1 + 0 - 0 = 0$ not true.

Q.5 (A, C)

$5 \sin^2 x + \sqrt{3} \sin x \cos x + 6 \cos^2 x = 5$

Case-I $\cos x = 0 \Rightarrow 5 + 0 + 0 = 5$

$\therefore x = n\pi + \frac{\pi}{2}$

Case -II $\cos x \neq 0$

$\therefore 5\tan^2 x + \sqrt{3} \tan x + 6 = 5(1 + \tan^2 x)$

$\Rightarrow \tan x = -\frac{1}{\sqrt{3}}$

Q.6 (A, B)

Let $E = \sin x - \cos^2 x - 1$

$\Rightarrow E = \sin x - 1 + \sin^2 x - 1 = \sin^2 x + \sin x - 2$

$= \left(\sin x + \frac{1}{2}\right)^2 - \frac{9}{4}$ assumes least value

when $\sin x = -\frac{1}{2} \Rightarrow x = n\pi + (-1)^n \left(-\frac{\pi}{6}\right)$.

Q.7 (A, B, C, D)

$2\sin\frac{x}{2} . \cos^2 x + \sin^2 x = 2 \sin\frac{x}{2} . \sin^2 x + \cos^2 x$

$\Rightarrow 2\sin\frac{x}{2} (\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\Rightarrow \sin 2x = 1$ or -1 αnd $\cos x = 1 - 2\sin^2\frac{x}{2} =$

$1 - 2\left(\frac{1}{4}\right) = \frac{1}{2}$

$\therefore \cos 2x = 2\cos^2 x - 1 = 2 \times \frac{1}{4} - 1 = -\frac{1}{2}$

Q.8 (B, C, D)

$2\sin 2x = \sin x + \sin 3x$

$\Rightarrow 2\sin 2x = 2\sin 2x \cos x$

$\Rightarrow \sin 2x = 0$ or $\cos x = 1$

$\Rightarrow 2x = n\pi$ or

$x = 2m\pi$

$\Rightarrow x = \frac{n\pi}{2}, 2m\pi$

options (A), (B), (C), (D) are all a part of $x = \frac{n\pi}{2}$.

Q.9 (B, D)

$\sin x + \sin 2x + \sin 3x = 0$

$\Rightarrow 2 \sin 2x \cos x + \sin 2x = 0$

$\Rightarrow \sin 2x = 0$ or $\cos x = \frac{-1}{2}$.

Q.10 (A, B, C)

$\cos 4x \cos 8x - \cos 5x \cos 9x = 0$

$\Rightarrow 2\cos 4x \cos 8x = 2\cos 5x \cos 9x$

$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x$

$\Rightarrow 14x = 2n\pi \pm (12x)$

$\Rightarrow 2x = 2n\pi$ or $26x = 2n\pi$

$\Rightarrow x = n\pi$ or $\frac{n\pi}{13}$

$\therefore \sin x = 0$ or $\sin 13x = 0$

Q.11 (A,C)

$\cos x \cos 6x = -1 \Rightarrow 2 \cos x \cdot \cos 6x = -2$

$\Rightarrow \cos 7x + \cos 5x = -2$

It is possible only when

$\cos 7x = -1$ & $\cos 5x = -1$

$7x = (2n + 1)\pi$ & $5x = (2m + 1)\pi$; n, m $\in$ I

$x = (2n + 1)\frac{\pi}{7}$ & $x = (2m + 1)\frac{\pi}{5}$

$x = \frac{\pi}{7}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}$ & $x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

common solution in one round is π

In general $x = (2n \pm 1)\pi$, $n \in I$

Q.12 (A, C)

$\cos x . \cos 6x = -1$

$\Rightarrow$ Either $\cos x = 1$ and $\cos 6x -1$

or $\cos x = -1$ and $\cos 6x = 1$

$\Rightarrow x = 2n\pi$ and $\cos 6x = -1$

or $x = (2n + 1)\pi$ and $\cos 6x = 1$

If $x = 2n\pi$ then $\cos 6x$ cannot be -1

However if $x = (2n + 1)\pi$ then $\cos 6x = 1$

$\therefore x = (2n + 1)\pi$

$x = (2n - 1)\pi$ is also as above.

Q.13 (B, C)

Given, $2 \cos t = \frac{3x+1}{x-1}$

So, we must have $-1 \leq \frac{3x+1}{2x-2} \leq 1$

$\therefore$ On solving, we get

$x \in \left[-3, \frac{1}{5}\right]$

Q.14 (B, C)
sin x + 2 cos k cos x = 2
for real solution(s)

$\sqrt{1+4\cos^2 k} \geq 2$

Hence sin k ∈ $\left[\frac{-1}{2}, \frac{1}{2}\right]$

Q.15 (B, D)
$\log_{10}(\sin x) + \log_{10}(\tan y) + \log_{10} 2 = 0$
2 sinx tan y = 1 ....(1) sinx > 0 , tan y > 0

cot y = $2\sqrt{3}$ cos x ....(2)

from (1) and (2) 2 sin x = $2\sqrt{3}$ cos x

∴ tan x = $\sqrt{3}$

⇒ $x = 2n\pi + \frac{\pi}{3}$, n ∈ I {∴ sin x > 0}

From (1) $\tan y = \frac{1}{\sqrt{3}}$ ⇒ $y = n\pi + \frac{\pi}{6}$, n ∈ I {∴ tan y > 0}

Q.16 (A, B,C)
(sin x –1) cos x ≥ 0
⇒ (1 – sin x) cos x ≤ 0
⇒ cos x ≤ 0

⇒ $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ ≃ [1.57, 4.71]

Comprehension # 1 (Q. No. 17 to 19)

Q.17 (D)
$4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = (2\sin x + 1)(2\sin^2 x - 1) = 0$

∴ $\sin x = -\frac{1}{2}, \pm\frac{1}{\sqrt{2}}$

∴ there are 6 solutions.

Q.18 (C)

$3 = \cos 4x + \frac{10\tan x}{1+\tan^2 x} = \cos 4x + 5\sin 2x$

i.e $3 = 1 - 2\sin^2 2x + 5\sin 2x$

i.e $\sin 2x = \frac{1}{2}$

∴ $2x = \frac{\pi}{6}, \frac{5\pi}{6}$

Thus there are two solutions.

Q.19 (B)
(i) when tan x ≥ 0, then the equation becomes $\tan x = \tan x + \frac{1}{\cos x}$ i.e $\frac{1}{\cos x} = 0$ (not possible)

(ii) when tan x < 0, then the equation becomes $-\tan x = \tan x + \frac{1}{\cos x}$ i.e $\sin x = -\frac{1}{2}$

∴ $x = \frac{11\pi}{6}$ is the only solution.

Comprehension # 2 (Q. No. 20 to 22)

Q.20 (A)
Given sin x + cos x = 1 + sin x cos x

$t = 1 + \frac{t^2-1}{2}$

$(t-1)^2 = 0$
t = 1
sin x + cos x = 1

so $x = 2n\pi$ and $2n\pi + \frac{\pi}{2}$

Q.21 (B)
(cos x – sin x) (2 sinx + 1) + 2 cos x = 0
$\sin x(2\cos x - 1) + 3\cos x - 2(1-\cos^2 x) = 0$
sin x(2 cos x – 1) + 2 cos x (cos x + 2)
– 1 (cos x + 2) = 0
⇒ (sin x + cos x + 2) (2 cos x – 1) = 0

∴ $\cos x = \frac{1}{2}$

⇒ $x = 2n\pi \pm \frac{\pi}{3}$

Q.22 (C)
Given $\sin^4 x + \cos^4 x = \sin x \cos x$

⇒ $1 - \frac{(t^2-1)^2}{2} = t^2 - 1$

⇒ $t^4 - t^2 - 2 = 0$
Let $t^2 = u$
$u^2 - u - 2 = 0$
u = 2 or u ≠ –1

∴ $t = \pm\sqrt{2}$

⇒ $\sin x + \cos x = \pm\sqrt{2}$
⇒ sin (x – π/4) = 1

⇒ $x = n\pi + \frac{\pi}{4}$

Comprehension # 3 (Q. No. 23 to 25)

Q.23 (D)
$3^{3\cos 2x} . 3^{4\sin 2x}$
$3^{3\cos 2x + 4\sin 2x}$

minimum value of 3 cos 2 x + 4 sin 2x is –5

so minimum value $= 3^{-5} = \frac{1}{243}$

Q.24 (D)

When $\sin^4 x = 1$, $\cos^7 x = 0$

so in [0, 2π], number of solution = 2

when $\cos^7 x = 1$, $\sin^4 x = 0$

So in [0, 2π], number of solution = 2

Total number of solution = 4

Q.25 (D)

Given cos (p sin x) = sin (p cos x)

$\Rightarrow \cos(p \sin x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\Rightarrow p \sin x = \frac{\pi}{2} - p\cos x$

$\Rightarrow p(\sin x + \cos x) = \frac{\pi}{2}$

then maximum value of (sin x + cos x) will give minimum positive value of p

so $p \times \sqrt{2} = \frac{\pi}{2}$, $p = \frac{\pi}{2\sqrt{2}}$

Comprehension # 4 (Q. No. 26 to 28)

Q.26 (C)

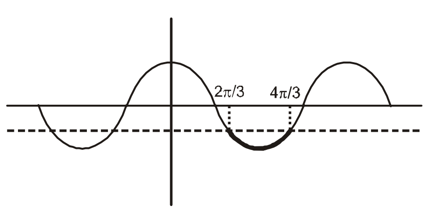
$\sin^6 x + \cos^6 x < \frac{7}{16} \Rightarrow 1 - 3\sin^2 x \cos^2 x < \frac{7}{16}$

$\Rightarrow \sin^2 x \cos^2 x > \frac{3}{16} \Rightarrow \sin^2 2x > \frac{3}{4}$

$\Rightarrow \frac{1 - \cos 4x}{2} > \frac{3}{4} \Rightarrow 1 - \cos 4x > \frac{3}{2}$

$\Rightarrow \cos 4x < -\frac{1}{2}$

$\Rightarrow$ Principal is value $4x \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$



$\Rightarrow$ General value is

$4x \in \left(2n\pi + \frac{2\pi}{3},\ 2n\pi + \frac{4\pi}{3}\right)$

$\Rightarrow x \in \left(\frac{n\pi}{2} + \frac{\pi}{6}, \frac{n\pi}{2} + \frac{\pi}{3}\right)$, $n \in I$

Q.27 (D)

$\cos 2x + 5\cos x + 3 \ge 0$

$\Rightarrow 2\cos^2 x + 5\cos x + 2 \ge 0$

$\Rightarrow (\cos x + 2)(2\cos x + 1) \ge 0$

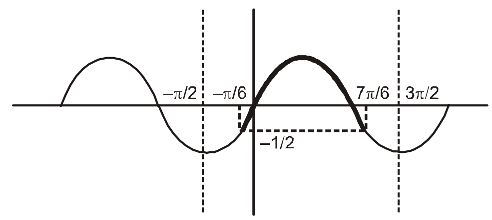
$\Rightarrow 2\cos x + 1 \ge 0$

$(\because \cos x + 2 > 0)$

$\Rightarrow \cos x \ge -\frac{1}{2} \Rightarrow x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$

Q.28 (D)

$2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \ge 0$



$\Rightarrow 1 - \cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3}\cos 2x \ge 0$

$\Rightarrow \sqrt{3}\cos 2x + \sin 2x \ge -1$

$\Rightarrow \frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x \ge -\frac{1}{2}$

$\Rightarrow \sin\left(2x + \frac{\pi}{3}\right) \ge -\frac{1}{2}$

$\Rightarrow 2x + \frac{\pi}{3} \in \left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{7\pi}{6}\right]$

$\Rightarrow 2x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}\right]$

$\Rightarrow x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{5\pi}{12}\right]$

$\Rightarrow x \in \left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$ in $[-\pi, \pi]$

Q.29 (A) → (q), (B) → (r), (C) → (p), (D) → (r)

(A) $\sin^2\theta = \sec^2\theta$

$\Rightarrow \sin^2\theta \cos^2\theta = 1$

$\Rightarrow \sin^2\theta(1 - \sin^2\theta) = 1$

$\Rightarrow \sin^4\theta - \sin^2\theta + 1 = 0$

Let $\sin\theta = t$

$\therefore$ $t^2 - t + 1 = 0$; $D = 1 - 4 = -3$
Since $D < 0$, there is no real solution
(B) $\sin\theta + \cos\theta = 1$
If $\sin\theta = 0$, then $\cos\theta = 1$
$\therefore$ In $[0, 2\pi]$, Number of solution = 2
and if $\sin\theta = 1$ then $\cos\theta = 0$
$\therefore$ In $[0, 2\pi]$, Number of solution = 1
Total number of solution = 3
(C) $\tan\theta + \sec\theta = 2\cos\theta$

$\Rightarrow \frac{1}{\cos\theta}[\sin\theta + 1] = 2\cos\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-1 \pm 3}{4}$

$\therefore$ 2 real solutions
(D) $3\sin^2\theta - 4\sin\theta + 1 = 0$
If $\sin\theta = 1$, In $[0, 2\pi]$ one solution

& $\sin\theta = \frac{1}{3}$, in $[0, 2\pi]$, there are 2 solutions

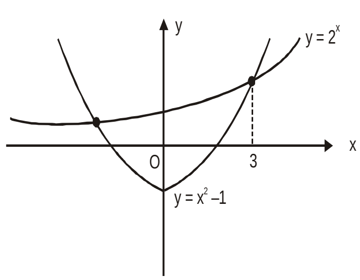
Total number of solution = 3

Q.30 (A) $\rightarrow$ (q), (B) $\rightarrow$ (s), (C) $\rightarrow$ (r), (D) $\rightarrow$ (p)
(A) $\sin^2\theta + 3\cos\theta = 3 \Rightarrow 1 - \cos^2\theta + 3\cos\theta = 3$
$\Rightarrow \cos^2\theta - 3\cos\theta + 2 = 0 \Rightarrow \cos\theta = 1, 2$
$\Rightarrow \cos\theta = 1$ $(\because \cos\theta \neq 2)$
$\Rightarrow \theta = 0$ in $[-\pi, \pi]$
$\therefore$ No. of solution = 1

(B) $\sin x \cdot \tan 4x = \cos x \Rightarrow \sin x.\frac{\sin 4x}{\cos 4x} = \cos x$

$\Rightarrow \sin 4x \sin x - \cos 4x \cos x = 0 \Rightarrow \cos 5x = 0$
$\Rightarrow 5x = (2n + 1)\pi/2 \Rightarrow x = (2n + 1)\pi/10$

$\Rightarrow x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$ in $(0, \pi)$



So there are five solutions.

(C) $(1 - \tan^2\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

$\Rightarrow (1 - \tan^4\theta) + 2^{\tan^2\theta} = 0$
$\Rightarrow (1 - x^2) + 2^x = 0$ where $x = \tan^2\theta$
$\Rightarrow 2^x = x^2 - 1 \Rightarrow x = 3$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3}$

$\Rightarrow \theta = \pm\frac{\pi}{3}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Number of solutions = 2

(D) $[\sin x] + [\sqrt{2}\cos x] = -3$

$\Rightarrow [\sin x] = -1$ and $[\sqrt{2}\cos x] = -2$

$\Rightarrow \pi < x < 2\pi$ and $-2 \le \sqrt{2}\cos x < -1$

$\Rightarrow -2 \le \cos x < -\frac{1}{\sqrt{2}}$

$\Rightarrow -1 \le \cos x < -\frac{1}{\sqrt{2}}$

$\Rightarrow \pi \le x < \frac{5\pi}{4}$ for

$x \in [0, 2\pi]$ $\pi \le x < \frac{5\pi}{4}$, $x \in [0, 2\pi]$

$\therefore \pi < x < \frac{5\pi}{4} \Rightarrow 2\pi < 2x < \frac{5\pi}{2}$

$\Rightarrow 0 < \sin 2x < 1 \Rightarrow [\sin 2x] = 0$

Q.31 **(A)$\rightarrow$(p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)**
(A) $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$
$\Rightarrow (2\sin x - \cos x)(1 + \cos x) = 1 - \cos^2 x \Rightarrow$
$(2\sin x - \cos x)(1 + \cos x)$
$= (1 - \cos x)(1 + \cos x)$
$\Rightarrow (1 + \cos x)(2\sin x - 1) = 0$

$\therefore \sin x = \frac{1}{2}$ or $\cos x = -1$

(B) $1 + \sin 2x = (\cos x + \sin x)$
$\Rightarrow (\sin x + \cos x)^2 = (\cos x + \sin x) \Rightarrow (\cos x + \sin x)(\cos x + \sin x - 1) = 0$
So, $\cos x + \sin x = 1$ & $\cos x + \sin x = 0$
$\therefore \tan x = -1$

(C) $4x^2 + x^6 + \sin^2 5x = 0$
$\sin^2 5x = -(4x^4 + x^6) \rightarrow$ always negative

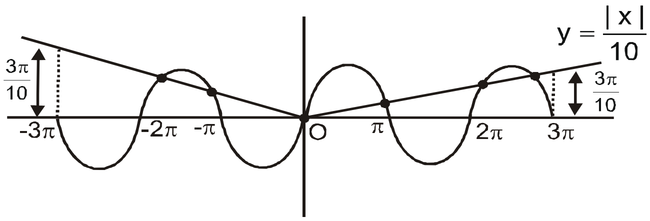
$\downarrow$
always positive
So solution is $x = 0$ since LHS is always positive and RHS is always negative.

(D) Given that, $\tan x = \frac{1}{\sqrt{3}} \Rightarrow x = n\pi \pm \frac{\pi}{6}$

So, solution is $\frac{19\pi}{6}$

Q.32 (A) → (r),(B) → (p, q, r, s), (C) → (q), (D) → (s)

(A)

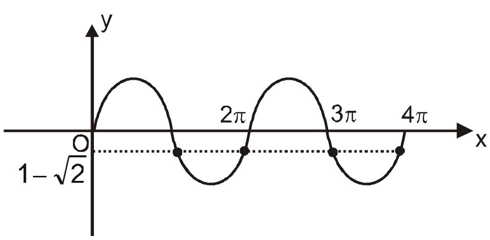


Number of solutions = 6

(B) $\sin x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

$\Rightarrow \sin x = 1 - \sqrt{2}$

As sin x takes at least four values



in [0, nπ] ∴ n ≥ 4

(C) $1 + \sin^4 x = \cos^2 3x$

L.H.S. ≥ 1 and R.H.S. ≤ 1

∴ L.H.S. = R.H.S. = 1 $\Rightarrow \sin^4 x = 0$ and $\cos^2 3x = 1$

$\Rightarrow x = n\pi$ and $3x = m\pi$

$\Rightarrow x = n\pi$ and $3x = m\pi$

$\Rightarrow x = n\pi$ and $x = \frac{m\pi}{3}$

$\Rightarrow x = n\pi$

$\Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$ in $\left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$

∴ Number of solutions = 5.

(D) A, B, C are in A.P. ⇒ B = 60º

As $\sin(2A + B) = \frac{1}{2} \Rightarrow 2A + B = 30º$ or $150º$

⇒ 2A = –30º or 90º ⇒ 2A = 90º ⇒ A = 45º

∴ $C = 180º - A - B = 75º = \frac{5\pi}{12}$ ∴ p = 12.

NUMERICAL VALUE BASED

Q.1 15

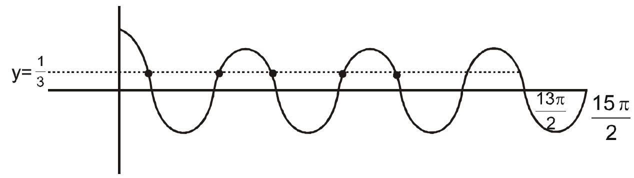
$2\tan^2 x - 5\sec x - 1 = 0$

$\Rightarrow 2(\sec^2 x - 1) - 5\sec x - 1 = 0$

$\Rightarrow 2\sec^2 x - 5\sec x - 3 = 0$

$\Rightarrow \sec x = \frac{6}{2}, \frac{-1}{2} = 3, \frac{-1}{2}$

$\Rightarrow \sec x = 3\left(\sec x \neq \frac{-1}{2}\right)$



$\Rightarrow \cos x = \frac{1}{3}$

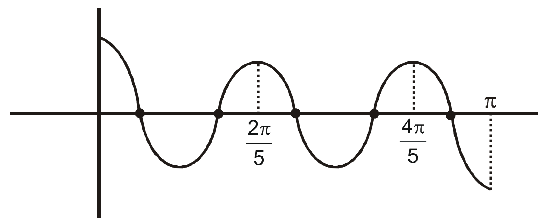
⇒ 7 solutions in $\left[0, \frac{15\pi}{2}\right]$

∴ n = 15.

Q.2 5

sin x . tan 4x = cos x ⇒ sin x sin 4x = cos x cos 4x

⇒ cos 5x = 0 ⇒ five solutions.



Q.3 5

sin 7x + sin 4x + sin x = 0

⇒ 2 sin 4x cos 3x + sin 4x = 0

$\Rightarrow \sin 4x = 0$ or $\cos 3x = -\frac{1}{2}$

$\Rightarrow 4x = n\pi$ or $3x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{4}, \frac{2n\pi}{3} \pm \frac{2\pi}{9} = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{9}, \frac{4\pi}{9}.$

Q.4 2

Given tan x + sec x = 2 cos x

or $\frac{\sin x + 1}{\cos x} = 2\cos x$

or $2\cos^2 x = 1 + \sin x$

or $2 - 2\sin^2 x = 1 + \sin x$

or $2\sin^2 x + \sin x - 1 = 0$

or $2\sin^2 x + 2\sin x - \sin x - 1 = 0$

or $(\sin x + 1)(2\sin x - 1) = 0$

⇒ either sin x = –1 or

$\sin x = \frac{1}{2}$

$\Rightarrow x = \frac{3\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\qquad [\because \; x \in (0, 2\pi)]$

but when $x = \frac{3\pi}{2}$, tan x and sec x are not defined.

$\therefore$ solutions of given equation are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ only

$\Rightarrow$ number of solutions = 2

Q.5 7

Given equation is

$\sin x\,[4(1 - \sin^2 x) - 2\sin x - 3] = 0$

$\therefore \; \sin x = 0$ or

$4\sin^2 x + 2\sin x - 1 = 0$

$\sin x = 0$ or

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$\sin x = 0$ or

$$\sin x = \frac{\sqrt{5} - 1}{4}$$

or $\sin x = \frac{\sqrt{5} + 1}{4}$

$\therefore$ General solution is

$x = n\pi$

or $x = n\pi + (-1)^n \frac{\pi}{10}$

or $x = n\pi + (-1)^n \left(-\frac{3\pi}{10}\right)$, $n \in Z$.

Forn = 1, 2

$$x = \frac{13\pi}{10}, \frac{17\pi}{10}.$$

Q.6 4

Given equation is

$1 + |\cos x| + |\cos x|^2 + \ldots\ldots = 2$

$\Rightarrow \frac{1}{1 - |\cos x|} = 2 \qquad \Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow \cos x = \pm\frac{1}{2} \qquad \Rightarrow \; x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$

Q.7 5

$$\cos 3x + \cos 2x = \sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)$$

$$2\cos\frac{5x}{2}\cos\frac{x}{2} = 2\sin(x)\cos\frac{x}{2}$$

$\Rightarrow \cos\frac{x}{2} = 0$ or

$\cos\frac{5x}{2} = \sin x$

$\Rightarrow \frac{x}{2} = (2n + 1)\frac{\pi}{2}$

$\frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x\right)$

$\Rightarrow \; x = (2n + 1)\pi$

Taking positive sign

$$\frac{7x}{2} = 2n\pi + \frac{\pi}{2}$$

$$x = (4n + 1)\frac{\pi}{7}$$

Taking negative sign

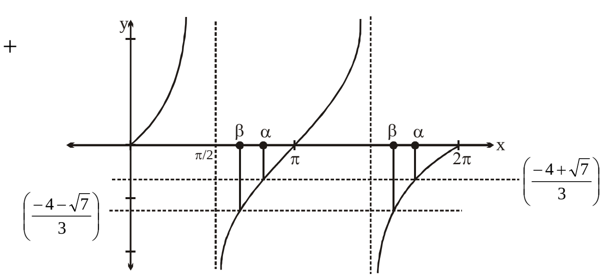
$\frac{3x}{2} = 2n\pi - \frac{\pi}{2} \quad \Rightarrow \quad x = (4n - 1)\frac{\pi}{3}$

$\therefore \; 0 \le x \le 2\pi$, $\qquad$ x

$= \pi, \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$

Q.8 5

+



$\tan(\pi - \alpha) = \frac{-4 + \sqrt{7}}{3}$,

$\tan(\pi - \beta) = \frac{-4 - \sqrt{7}}{3}$

$\tan\alpha = 4 - \frac{\sqrt{7}}{3}$, $\qquad \tan\beta = 4 + \frac{\sqrt{7}}{3}$

$\sum x = (\pi - \alpha) + (\pi - \beta) + (2\pi - \alpha) + (2\pi - \beta) = 6\pi$

$- 2(\alpha + \beta)$ (Where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$)

Now tan α + β = $\dfrac{\frac{8}{3}}{1-\frac{9}{9}} = \infty \Rightarrow$ Means $\alpha + \beta = \dfrac{\pi}{2}$

$(\because \alpha+\beta \in (0,\pi))$

Therefore $\sum x = 6\pi - 2(\alpha + \beta)$

$\Rightarrow \sum x = 6\pi - 2\left(\dfrac{\pi}{2}\right) \Rightarrow 5\pi$

Hence the value of k = 5. **Ans.**

KVPY
PREVIOUS YEAR'S

Q.1 (A)

$x^2 + 2x \sin(xy) + 1 = 0$

$2\sin(xy) = -\left(1+\dfrac{1}{x}\right)$

R.H.S. ≥ 2 or ≤ – 2

⇓ ⇓

L.H.S. = R. H.S = 2 Sin (xy) = – 1

⇓ (x = 1)

x = – 1

sin (–y) = 1   sin y = –1

sin y = – 1

$y = 2n\pi - \dfrac{\pi}{2}, n \in I$

Hence pair of straight lines.

Q.2 (A)

Square & add both equations

9 + 16 + 24 sin (A + B) = 37

$\sin(A + B) = \dfrac{1}{2} \Rightarrow A + B = \dfrac{\pi}{6} \Rightarrow C = \dfrac{5\pi}{6}$ (wrong)

$\Rightarrow A+B=\dfrac{5\pi}{6} \Rightarrow C=\dfrac{\pi}{6}$

because $C = \dfrac{5\pi}{6}$

does not follow equation 3 sin A + 4 cos B = 6 {x}

Q.3 (C)

$\sin x + \dfrac{1}{2}\cos x = \sin^2\left(x+\dfrac{\pi}{4}\right)$

$\sin x + \dfrac{1}{2}\cos x = \dfrac{1}{2}\left(1-\cos\left(\dfrac{\pi}{2}+2x\right)\right)$

$\sin x + \dfrac{1}{2}\cos x = \dfrac{1}{2}(1 + \sin 2x)$

2 sin x + cos x =1 + sin x cos x

2 sin x cos x – 2sin x + 1 – cos x = 0

(1 – cos x) – 2 sin x (1 – cos x) = 0

(1 – cos x) (1 – 2 sin x) = 0

1 – cos x = 0   1 – 2 sin x = 0

cos x = 1 $\sin x = \dfrac{1}{2}$

x = 0, $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$\text{sum} = 0 + \dfrac{\pi}{6}, \dfrac{5\pi}{6} = \pi$

Q.4 (D)

$2\sin\alpha + 3\cos\beta = 3\sqrt{2}$ ... (A)

$3\sin\beta + 2\cos\alpha = 1$ ... (B)

sum of squares of equation (A) and (B)

4 + 9 + 12 sin (α + β) = 19

$\sin(\alpha + \beta) = \dfrac{1}{2} \Rightarrow \alpha + \beta = 150°$ or $30°$

If α + β = 30° ⇒ β = 30 – α

put in equation (A) and (B)

we get $7\sin\alpha + 3\sqrt{3}\cos a = 6\sqrt{2}$

$7\cos\alpha - 3\sqrt{3}\sin\alpha = 2$

$\cos\alpha = \dfrac{7+9\sqrt{6}}{37} = .8 < \dfrac{\sqrt{3}}{2}$

cos a < cos 30º ∵ a > 30º   ∴ α + β ≠ 30º

Ans : (D)

Q.5 (B)

$\cos^4 x - \sin^4 x = \dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x}$

$(\cos^2 x - \sin^2 x) = \dfrac{(\cos^2 x - \sin^2 x)}{\sin^2 x\cos^2 x}$

$\cos 2x = \dfrac{4\cos 2x}{\sin^2 2x}$

$\cos 2x\left(1 - 4\,\mathrm{cosec}^2 2x\right) = 0$

cos 2x = 0

$2x = 2n\pi \pm \dfrac{\pi}{2}$

$x = n\pi \pm \dfrac{\pi}{4}$

At $n = 0,\ x = \dfrac{\pi}{4}$

$n = 1;\ x = \dfrac{5\pi}{4}, \dfrac{3\pi}{4}$

$n = 2, x = \dfrac{7\pi}{4}$

Q.6 (B)

$\dfrac{\sin(\lambda\alpha)}{\sin\alpha} - \dfrac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$

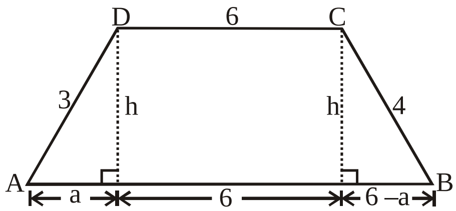
By observation

$\sin(\lambda\alpha)\cos\alpha - \cos(\lambda\alpha)\sin\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

$\sin(\lambda - 1)\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

clearly $\lambda = 1$, $\lambda = 3$ is solution

Q.7 (B)

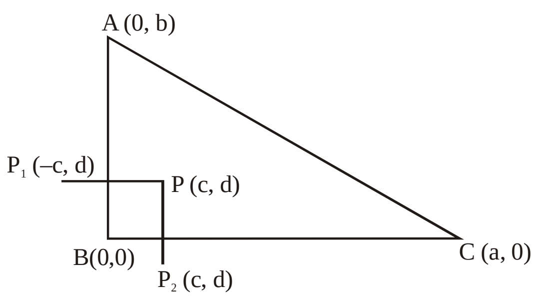


Solve $a^2 + h^2 = 9$ ....(A)

and $(6 - a)^2 + h^2 = 16$ ....(B)

we will get h = 2.4

Q.8 (C)



M is circumcentre of ΔABC

$\Rightarrow M\left(\frac{a}{2}, \frac{b}{2}\right)$

& N is circumcentre of ΔABC

$N = (0, 0) = B$ (Mid-point of P_1, & P_2).

So $MN = \frac{AC}{2}$

Q.9 (A)

$\cos(\sin x) = \sin(\cos x)$

$$\sin\left(\frac{\pi}{2} \pm \sin x\right) = \sin(\cos x)$$

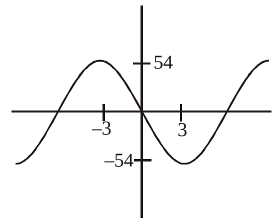
$$\cos x = n\pi + (-1)^n\left(\frac{\pi}{2} + \sin x\right), n \in I$$

$$\Rightarrow \cos x \pm \sin x = n\pi(-1)^n \frac{\pi}{2}, n \in I$$

As LHS $\in \left[-\sqrt{2}, \sqrt{2}\right]$, and it does not satisfies RHS

So No solution possible

Q.10 (B)



Let $f(x) = x^3 - 27$

$f'(x) = 3x^2 - 27 = 3(x^2 - 9)$

As sum of the roots is zero, so if two roots are integer then 3rd root has to be Integer

Now put x = 6t

$216t^3 - 27 \times 6t + k = 0$

$54(4t^3 - 3t) + k = 0$

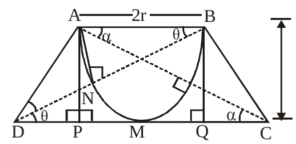
Put $t = \cos\theta$

$54\cos 3\theta = -k$

Now for $3\theta - 0, 2\pi$ we get integral solution

So two values of 'k'

Q.11 (D)



Join AN

$\because \angle ANB = 90^o$

In ΔANB,

$$\cos\theta = \frac{BN}{2r}$$

$BN = 2r\cos\theta$

$BD = 2BN = 4r\cos\alpha$

In ΔBQD

$$\sin\theta = \frac{BQ}{BD} = \frac{r}{4r\cos\theta}$$

$$\sin 2\theta = \frac{1}{2}$$

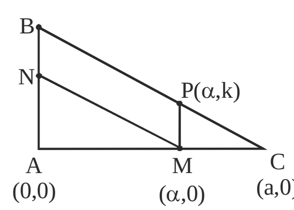
$\theta = 15^o$

Now similarly $\alpha = 15^o = \theta$ & $AC = 4r\cos\alpha$

$\therefore$ Trapezium will be isosceles

$\because \angle ADB = 30^o$

Q.12 (A)



To find $\frac{AM}{MC} = \frac{\alpha}{a - \alpha}$ ....(i)

$\therefore \Delta BAC \sim \Delta PMC$

$\frac{PM}{CM} = \frac{BA}{AC}$

$\frac{k}{a-\alpha} = \frac{BA}{a}$

$BA = \frac{ak}{a-\alpha}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}AC.\ AB = \frac{1}{2}a.\frac{ak}{a-\alpha}$ ......(ii)

Area of quad. BNMP = Ar trap. AMPB – ar ΔAMN ....(iii)

Also $\Delta NAM \sim \Delta BAC$

$\frac{AN}{AM} = \frac{AB}{AC}$

$\frac{AN}{\alpha} = \frac{ak}{(a-\alpha)a}$

$A = \frac{k\alpha}{a-\alpha}$

Put in (iii)

Area quad. BNMP = $\frac{1}{2}(AB + PM) \times AM - \frac{1}{2}AM \times AN$

$= \frac{1}{2}\left|\frac{ak}{a-\alpha}+k\right| \times \alpha - \frac{1}{2}\alpha \times \frac{k\alpha}{a-\alpha}$

$= \frac{1}{2}\left[\frac{2ak\alpha - k\alpha^2 - k\alpha^2}{a-\alpha}\right]$

$= \frac{ak\alpha - k\alpha^2}{a-\alpha}$

$= k\alpha$

Given $\alpha k = \frac{5}{18}\cdot\frac{1}{2}\cdot\frac{a^2k}{a-\alpha}$

$\frac{36}{5} = \frac{a^2}{\alpha(a-\alpha)}$

$36a\alpha - 36\alpha^2 = 5a^2$

$5a^2 - 36a\alpha + 36\alpha^2 = 0$

$5a^2 - 30a\alpha - 6a\alpha + 36\alpha^2 = 0$

$5a(a - 6\alpha) - 6\alpha(a - 6\alpha) = 0$

$5a = 6\alpha$ or $a = 6\alpha$

Not true as M is near to C than A

$\therefore \quad \frac{\alpha}{a-\alpha} = \frac{1}{\frac{a}{\alpha}-1} = \frac{1}{\frac{6}{5}-1} = 5$

Q.13 (A)

Make graph and observe yourself.

Q.14 (B)

$6\sin\theta - 2\sin 3\theta - 7\sin\theta + \sqrt{3}\cos\theta = 0$

$\sqrt{3}\cos\theta - \sin\theta = 2\sin 3\theta$

It can be written as

$2(\sin(60^\circ - \theta)) = 2\sin 3\theta$

$\sin(60^\circ - \theta) = \sin 3\theta$

$60^\circ = 4\theta$

$\Rightarrow \theta = 15^\circ$ is one of the value

Q.15 (B)

only possible when sin 3x = 1 & sin 7x = 1

$\sin 3x = 1$

$\sin 3x = \sin(4n + 1)\frac{\pi}{2}, n \in I$

$3x = (4n + 1)\frac{\pi}{2} \Rightarrow x = (4n + 1)\frac{\pi}{6}$

$\sin 7x = \sin(4m + 1)\frac{\pi}{2}, m \in I$

$x = (4m + 1)\frac{\pi}{14}$

For common solution

$(4n + 1)\frac{\pi}{6} = (4m + 1)\frac{\pi}{14}$

Solving these 1 = 3m – 7n

First solution is m = 5, n = 2

Second solution is m = 12, n = 5

So two solutions are possible.

Q.16 (B)

$\cos^2(x \sin 2x) + \frac{1}{1+x^2} = \cos^2 x + \sec^2 x$

LHS ≤ 2 RHS ≥ 2

LHS = RHS = 2 $\Rightarrow$ x = 0 only solutions

Q.17 (C)

$2\cos\left(x + \frac{2x^2}{2}\right).\sin\frac{x}{2} = 2\sin\frac{x}{2}.\cos\frac{x}{2}$

$\Rightarrow \sin\frac{x}{2}\left[\cos\left(\frac{x+2x^2}{2}\right) = \cos\frac{x}{2}\right] = 0$

$\sin\frac{x}{2} = 0$ or $2\sin\left(\frac{2x+2x^2}{4}\right).\sin\left(\frac{2x^2}{4}\right) = 0$

$\frac{x}{2} = 0, \pi, 2\pi$ or $\sin\left(\frac{x+x^2}{2}\right) = 0$ or $\sin\frac{x^2}{2} = 0$

$X = 0, 2\pi, 4\pi$ or $\sin\left(\frac{x+x^2}{2}\right) = 0, \pi, 2\pi$

$\frac{x+x^2}{2} = \pi$ $x^2 = 2\pi$

$\Rightarrow x^2 + x - 2\pi = 0$ $\Rightarrow x = \sqrt{2\pi}$

$X = \frac{-1 \pm \sqrt{1+8\pi}}{2}$ $\qquad 2 > \sqrt{2\pi} < 3$

$\sqrt{1+8\pi} \approx \sqrt{25.14}$

≈ 5.2

$\therefore \quad x = \frac{5.2-1}{2} = \frac{4.2}{2} = 2.1$

$\therefore$ total number of solution lies between (2, 3) = 2

Q.18 (B)

$$\frac{\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta}{9} = \frac{\sin^2\theta + 2\cos^2\theta}{3}$$

$\Rightarrow 2\sin^2\theta + 2\cos^2\theta - 4\sin\theta\cos\theta = 0$

$\Rightarrow \sin 2\theta = 1 \quad \Rightarrow \quad 2\theta = 2n\pi + \frac{\pi}{2} \Rightarrow \theta = n\pi + \frac{\pi}{4}$

$\Rightarrow A \cap [0, \pi] = \left\{\frac{\pi}{4}\right\}$

Q.19 (B)

$\sin 9x + \sin 3x = 0$

$\Rightarrow 2\sin 6x \cos 3x = 0$

$\Rightarrow 4\sin 3x \cos^2 3x = 0$

$\Rightarrow \sin 3x = 0$ or $\cos 3x = 0$

$\Rightarrow 7 + 6 = 13$ solutions

Q.20 (D)

$\sin(\pi - \pi\cos^2\theta) + \sin(\pi\cos^2\theta) = 2\cos\left(\frac{\pi\cos\theta}{2}\right)$

$\Rightarrow 2\sin(\pi\cos^2\theta) = 2\cos\left(\frac{\pi\cos\theta}{2}\right)$

$\Rightarrow \cos\left(\frac{\pi}{2} - \pi\cos^2\theta\right) = \cos\left(\frac{\pi\cos\theta}{2}\right)$

$\Rightarrow \frac{\pi}{2} - \pi\cos^2\theta = 2n\pi \pm \frac{\pi\cos\theta}{2}$

$\Rightarrow \frac{1}{2} - \cos^2\theta = 2n \pm \frac{\cos\theta}{2}$

$\Rightarrow 1 - 2\cos^2\theta = 4n \pm \cos\theta$

$\Rightarrow 2\cos^2\theta \pm \cos\theta = 4k + 1$

$\Rightarrow 2\cos^2\theta + \cos\theta = 1,\ 2\cos^2\theta - \cos\theta = 1$

$\Rightarrow \cos\theta = -1, \frac{1}{2}, 1, -\frac{1}{2} \quad \Rightarrow$ 7 solutions

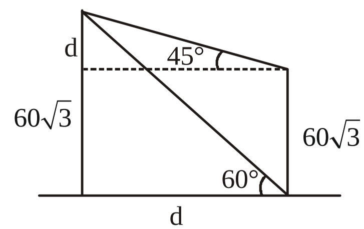
Q.21 (C)

$\frac{60\sqrt{3}+d}{d} = \sqrt{3}$

$\Rightarrow d = \frac{60\sqrt{3}}{\sqrt{3}-1}$

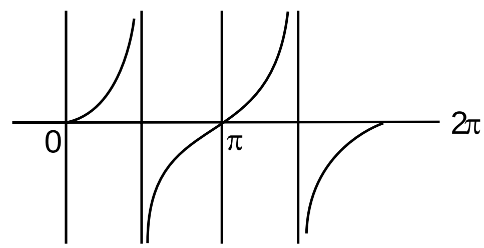
$\Rightarrow h = 60\sqrt{3}\left(1 + \frac{1}{\sqrt{3}-1}\right)$

$= \frac{60\times 3}{\sqrt{3}-1} = 90(\sqrt{3}-1)$



JEE MAIN-2021
PREVIOUS YEAR'S

Q.1 (1)



$\tan 2\theta\ (1 + \cos 2\theta) > 0$

$2\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Q.2 (3)

$x = y = \frac{\pi}{3}$ satisfy the equation

$\therefore \quad \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

Q.3 1

$\sqrt{3}\cos^2 x - (\sqrt{3} - 1)\cos x - 1 = 0$

$\cos x = \frac{(\sqrt{3}-1) \pm \sqrt{(\sqrt{3}-1)^2 + 4\sqrt{3}}}{2\sqrt{3}}$

$= \dfrac{(\sqrt{3}-1) \pm \sqrt{(4+2\sqrt{3}}}{2\sqrt{3}} = \dfrac{(\sqrt{3}\ -1) \pm\ (\sqrt{3}+1)}{2\sqrt{3}}$

$= 1, \dfrac{-1}{\sqrt{3}}$

since $x \in \left[0, \dfrac{\pi}{2}\right]$

$\Rightarrow \cos x = \dfrac{-1}{\sqrt{3}}$, not possible

$\therefore$ cos x = 1
$\Rightarrow$ x = 0
$\therefore$ number of solution 1

Q.4 (1)

$e^{(\cos^2\theta + \cos^4\theta + ...\infty)\ \ln 2} = 2^{\cos^2\theta + \cos^4\theta + ...\infty}$

$= 2^{\cot^2\theta}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$

$0 < \theta < \dfrac{\pi}{2} \Rightarrow \cot^2\theta = \sqrt{3}$

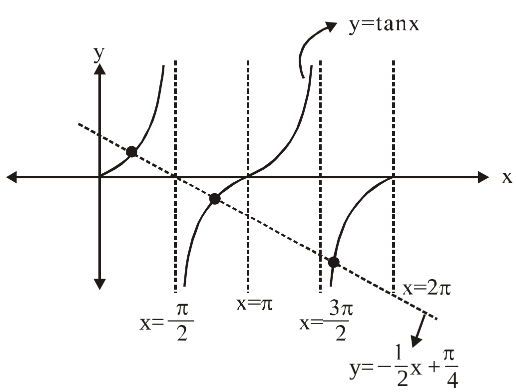
$\dfrac{2\sin\theta}{\sin\theta + \sqrt{3}\cos\theta} = \dfrac{2}{1+\sqrt{3}\sin\theta} = \dfrac{2}{4} = \dfrac{1}{2}$

Q.5 **(1)**

$x + 2\tan x = \dfrac{\pi}{2}$

$\Rightarrow 2\tan x = \dfrac{\pi}{2} - x$

$\Rightarrow \tan x = -\dfrac{1}{2} + \dfrac{\pi}{4}$



Number of soluitons of the given eauation is '3'.

Ans. (1)

Q.6 If $\cot x > 0 \Rightarrow \dfrac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \dfrac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$ (reject)

Q.7 **(2)**

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \dfrac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \dfrac{81}{t} = 30$

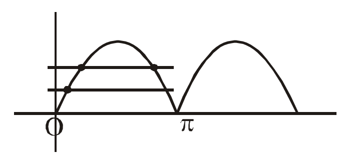
t2 − 30t + 81 = 0

(t − 3)(t − 27) = 0

$(81)^{\sin^2 x} = 3^1$ or $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$ or $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \dfrac{1}{4}$ or $\sin^2 x = \dfrac{3}{4}$



Total sol. = 4

Q.8 **(3)**
Q.9 (4)
Q.10 (1)
Q.11 **(2)**
Q.12 (3)
Q.13 (1)

$\dfrac{\cos x}{1+\sin x} = |\tan 2x|$

$\Rightarrow \dfrac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$

$\Rightarrow \tan^2\left(\dfrac{x}{4} - \dfrac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\dfrac{x}{4} - \dfrac{x}{2}\right)$

$\Rightarrow x = \dfrac{-3x}{10}, \dfrac{-x}{6}, \dfrac{\pi}{10}$

or sum $= \dfrac{-11\pi}{6}$.

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (D)

$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2} + 1)\cos\theta$

$\Rightarrow \tan\theta = \sqrt{2} + 1$

$\Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\therefore \cos\theta = (\sqrt{2} - 1)\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$

$\Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$\therefore P = Q$

Q.2 (D)

$\sin x + 2\sin 2x - \sin 3x = 3.$

$\sin x(1 + 2\cos x - 3 + 4\sin^2 x) = 3.$

$(4\sin^2 x + 2\cos x - 2) = \frac{3}{\sin x}$

$2 - 4\cos^2 x + 2\cos x = \frac{3}{\sin x}$

$\frac{9}{4} - \left(2\cos x - \frac{1}{2}\right)^2 = \frac{3}{\sin x}$.

L.H.S. $\le \frac{9}{4}$ R.H.S. ≥ 3.

No solution.

Q.3 [8]

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$

$\Rightarrow \cos^2 2x = \sin^2 2x$

$\Rightarrow \tan^2 2x = 1$

Now $2x \in [0, 4\pi]$ $\Rightarrow x =$

$\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

so number of solution

Q.4 (C)

$\sqrt{3}\sin x + \cos x = 2\cos 2x$

$\Rightarrow \cos 2x = \cos(x - \pi/3)$

$\Rightarrow x = 2n\pi - \pi/3$

or $1/3(2n\pi + \pi/3)$

$\therefore x = -\pi/3, -5\pi/9, \pi/9, 7\pi/9$

Hence, (C)

Q.5 (C)

$x = \sec\theta \pm \tan\theta$

$x = -\tan\theta \pm \sec\theta$

$\alpha_1 = \sec\theta - \tan\theta$

$\beta_2 = -\tan\theta - \sec\theta$

Hence, $\alpha_1 + \beta_2 = -2\tan\theta$

Hence, (C)

Q.6 (C)

$$\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)} = \frac{1}{\sin\frac{\pi}{6}}\sum_{k=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\right]}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$= 2\sum_{k=1}^{13}\left[\cot\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right]$$

$$= \left[\begin{aligned}&\left\{\cot\left(\frac{\pi}{4}\right)-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right\}+\left\{\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)\right\}\\&+\ldots\ldots+\left\{\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right\}\end{aligned}\right]$$

$$2\left[1-\cot\left(\frac{29\pi}{12}\right)\right] = 2\left[1-\cot\left(\frac{5\pi}{12}\right)\right] = 2\sqrt{3}-1$$

Q.7 **[0.5]**

$\sqrt{3}\cos x + \frac{2b}{a}\sin x = \frac{c}{a}$

Now, $\sqrt{3}\cos\alpha + \frac{2b}{a}\sin\alpha = \frac{c}{a}$ ....(1)

$\sqrt{3}\cos\beta + \frac{2b}{a}\sin\beta = \frac{c}{a}$ ....(2)

$\sqrt{3}[\cos\alpha - \cos\beta] + \frac{2b}{a}(\sin\alpha - \sin\beta) = 0$

$$\sqrt{3}\left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]+\frac{2b}{a}\left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]=0$$

$-\sqrt{3} + 2\sqrt{3}.\frac{b}{a} = 0$

$\frac{b}{a} = \frac{1}{2} = 0.5$

Q.8 (C)

Q.9 (2)

$f(x) = \sin(\pi\cos x)$

$X : \{x : f(x) = 0\}$

$f(x) = 0 \Rightarrow \sin(\pi\cos x)\ x = n \Rightarrow \cos = 1, -1.0 \Rightarrow x =$

$\frac{n\pi}{2}$

$X = \left\{\frac{n\pi}{2} : n \in N\right\} = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi.....\right\}$

g(x) = cos (2π sin x)
Z = {x : g(x) = 0}

$\cos(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = (2n+1)\frac{\pi}{2} \Rightarrow \sin x$

$= \frac{(2n+1)}{4}$

$\sin x = -\frac{1}{4}, \frac{1}{4}, \frac{-3}{4}, \frac{3}{4}$

$Z = \left\{n\pi \pm \sin^{-1}\left(\frac{1}{4}\right), n\pi \pm \sin^{-1}\left(\frac{3}{4}\right), n \in I\right\}$

Y = {X : f′(x) = 0}

$f(x) = \sin(\pi \cos x) = 0 \Rightarrow \pi \cos x = (2n+1)\frac{\pi}{2} \Rightarrow$

$\cos x = \frac{(2n+1)}{2} \Rightarrow \cos x = -\frac{1}{2}, \frac{1}{2}$

$Y = \left\{n\pi, n\pi \pm \frac{\pi}{3}\right\} =$

$\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi......\right\}$

W = {x : g′ (x) = 0}
g(x) = cos(2π sin x) ⇒ g′(x) = – sin (2π sin x). (2π cos x) = 0

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\sin(2\pi \sin x) = 0 \Rightarrow 2p \sin x = np \Rightarrow \sin x = \frac{n}{2} = -1$

$, -\frac{1}{2}, 0, \frac{1}{2}, 1$

$W = \left\{\frac{n\pi}{2}, n\pi \pm n \in 1\right\} =$

$\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, ...\right\}$

Now check the options

Q.10 [1.00]

Let $\pi x - \frac{\pi}{4} = \theta \in \left[\frac{-\pi}{4}, \frac{7\pi}{4}\right]$

So, $\left(3 - \sin\left(\frac{\pi}{2} + 2\theta\right)\right)\sin\theta \geq \sin(\pi + 3\theta)$

$\Rightarrow (3 - \cos 2\theta)\sin\theta \geq -\sin 3\theta$

$\Rightarrow \sin\theta[3 - 4\sin^2\theta + 3 - \cos 2\theta] \geq 0$
$\Rightarrow \sin\theta(6 - 2(1 - \cos 2\theta) - \cos 2\theta) \geq 0$
$\Rightarrow \sin\theta(4 + \cos 2\theta) \geq 0$
$\Rightarrow \sin\theta \geq 0$

$\Rightarrow \theta \in [0, \pi] \Rightarrow 0 \leq \pi x - \frac{\pi}{4} \leq \pi$

$\Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4}\right]$

$\Rightarrow \beta - \alpha = 1$

Solution of a Triangle

EXERCISES

ELEMENTRY

Q.1 (3)

$\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$

$\therefore\ \sin(B+A)\sin(B-A) = \sin(C+B)\sin(C-B)$

or $\sin C(\sin B\cos A - \cos B\sin A)$

$= \sin A(\sin C\cos B - \cos C\sin B)$

Divide by $\sin A\sin B\sin C$

$\therefore\ \cot A - \cot B = \cot B - \cot C$. Hence the result.

Q.2 (1)

$a\sin(B-C) + b\sin(C-A) + c\sin(A-B)$

$= k(\Sigma\sin A\sin(B-C)\ = k\{\Sigma\sin(B+C)\sin(B-C)\}$

$= k\left\{\Sigma\frac{1}{2}(\cos 2C - \cos 2B)\right\} = 0$

Q.3 (2)

$\sqrt{\frac{b+c}{4c}} = \sqrt{\frac{\sin 3C + \sin C}{4\sin C}} \Rightarrow \sqrt{\frac{2\sin 2C\cos C}{4\sin C}} = \cos C$

$\frac{b-c}{2c} = \frac{\sin 3C - \sin C}{2\sin C} = \frac{2\cos 2C\sin C}{2\sin C} = \cos 2C = \sin\frac{A}{2}$

Q.4 (4)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{5}{\sin\left(\frac{\pi}{2}+B\right)} = \frac{4}{\sin B}$

$\frac{5}{\cos B} = \frac{4}{\sin B},\ \therefore\ \tan B = \frac{4}{5}$

$\tan A = \tan\left(\frac{\pi}{2}+B\right) = -\cot B = \frac{-5}{4}3$

$\tan C = \tan(\pi - (A+B))\ = -\tan(A+B),\ [A+B+C=\pi]$

$= -\frac{(\tan A + \tan B)}{1 - \tan A.\tan B} = = \frac{-\left(-\frac{5}{4}+\frac{4}{5}\right)}{1+1} = \frac{9}{40}$

$C = \tan^{-1}\left(\frac{\left(2.\frac{1}{9}\right)}{1-\left(\frac{1}{9}\right)^2}\right),\ \therefore\ C = 2\tan^{-1}\left(\frac{1}{9}\right)$

Q.5 (2)

A, *B*, *C* are in A. P. then angle $B = 60^o$,

$\cos B = \frac{a^2+c^2-b^2}{2ac},\ \left\{\begin{matrix}\text{since } A+B+C=180^o \text{ and} \\ A+C=2B \Rightarrow B=60^o\end{matrix}\right\}$

$\Rightarrow \frac{1}{2} = \frac{a^2+c^2-b^2}{2ac} \Rightarrow a^2+c^2-b^2 = ac$

$\Rightarrow\ b^2 = a^2+c^2-ac$

Q.6 (1)

$(b^2-c^2)\cot A = (b^2-c^2)\frac{\cos A}{\sin A} = \frac{(b^2-c^2)(b^2+c^2-a^2)}{2bc.ka}$

Hence L.H.S. $= \frac{1}{2k\ abc}$

$[(b^4-c^4)+(c^4-a^4)+(a^4-b^4)-\{a^2(b^2-c^2)$

$+b^2(c^2-a^2)+c^2(a^2-b^2)\}] = 0$.

Q.7 (2)

Largest side is $\sqrt{p^2+pq+q^2}$. If largest angle is θ,

then $\cos\theta = \frac{p^2+q^2-p^2-pq-q^2}{2pq} = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow\ \theta = \frac{2\pi}{3}$

Q.8 (2)

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \lambda$ (Let)

$\therefore b+c = 11\lambda$ ….(i)

$c+a = 12\lambda$ ….(ii)

and $a+b = 13\lambda$ ….(iii)

From (i) + (ii) + (iii), $2(a+b+c) = 36\lambda$

$\therefore a+b+c = 18\lambda$

Now, (iv) – (i) gives, a= 7λ

(iv) – (ii) gives, b = 6 λ

(iv) – (iii) gives, c = 5λ

Q.9 (4)

We have, $b = \sqrt{3}, c = 1, \angle A = 30^o$

$\cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow \frac{\sqrt{3}}{2} = \frac{(\sqrt{3})^2+1^2-a^2}{2.\sqrt{3}.1}$

$\therefore\ a = 1,\ b = \sqrt{3},\ c = 1$

b is the largest side. Therefore, the largest angle *B* is given by $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{1+1-3}{2.1.1} = -\frac{1}{2}$

$\therefore\ B = 120°$

Q.10 (2)

$2ac \sin\frac{A-B+C}{2} = 2ac\sin\frac{\pi-2B}{2} = 2ac\cos B$

$2ac\frac{c^2+a^2-b^2}{2ca} = c^2+a^2-b^2$

Q.11 (2)

$\cos A = \frac{\sin B}{2\sin C} \Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{b}{2c}$

$\Rightarrow b^2+c^2-a^2-b^2 = 0 \Rightarrow c^2 = a^2$

Q.12 (1)

$a^4+b^4+c^4-2a^2c^2-2b^2c^2+2a^2b^2 = 2a^2b^2$

$\Rightarrow$

$(a^2+b^2-c^2)^2 = (\sqrt{2}ab)^2 \Rightarrow a^2+b^2-c^2 = \pm\sqrt{2}ab$

$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \pm\frac{\sqrt{2}ab}{2ab} = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \cos C = \cos 45^o$ or $\cos 135^o \Rightarrow C = 45^o$ or 135^o

Q.13 (1)

From the given relation $\sin C = \frac{1-\cos A\cos B}{\sin A\sin B} \le 1$ …..(i)

$\Rightarrow 1 \le \cos A\cos B + \sin A\sin B$

$\Rightarrow \cos(A-B) \ge 1$; $\because \cos\theta \not> 1$ …..(ii)

$\therefore\ A-B = 0$ or $A = B$

Hence from (i), $\sin C = \frac{1-\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$

$\therefore\ C = 90^o \Rightarrow A+B = 90^o$ or $A = B = 45^o$ {by (ii)}

Hence,

$a : b : c = \sin A : \sin B : \sin C = 1 : 1 : \sqrt{2}$.

Q.14 (4)

We have, $\frac{\tan\left(\frac{B}{2}\right)}{\cot\left(\frac{C-A}{2}\right)} = \frac{\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)}{\cos\frac{B}{2}\cos\left(\frac{C-A}{2}\right)}$

$= \frac{\sin C - \sin A}{\sin C + \sin A} = \frac{kc-ka}{kc+ka} = \frac{c-a}{c+a} = \frac{a}{3a} = \frac{1}{3}, \{\because c = 2a\}$

Q.15 (3)

$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{a^2-(b^2-c^2)}{2ac}$

Now $AD = \frac{abc}{b^2-c^2}$ $\therefore$ $\cos B = \frac{a^2-\frac{abc}{AD}}{2ac}$

Also, $AD = b\sin 23^o$ $\therefore$ $\cos B = \frac{a-\frac{c}{\sin 23^o}}{2c}$

By sine formula, $\frac{a}{c} = \frac{\sin(B+23^o)}{\sin 23^o}$

$\therefore\ \cos B = \left(\frac{\sin(B+23^o)}{\sin 23^o} - \frac{1}{\sin 23^o}\right) \div 2$

$\Rightarrow \sin(23^o - B) = -1 = \sin(-90^o)$

$\Rightarrow 23^o - B = -90^o$ or $B = 113^o$

Q.16 (2)

We have $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\ b^2 - 2bc\cos A + (c^2-a^2) = 0$

It is given that b_1 and b_2 are roots of this equation.

Therefore $b_1+b_2 = 2c\cos A$ and $b_1b_2 = c^2-a^2$

$\Rightarrow 3b_1 = 2c\cos A$, $2b_1^2 = c^2-a^2$ $(\because b_2 = 2b_1$ (given)

$2\left(\frac{2c}{3}\cos A\right)^2 = c^2-a^2 \Rightarrow 8c^2(1-\sin^2 A) = 9c^2-9a^2$

Q.17 (3)

$\frac{b-c}{a} = \frac{\sin B - \sin C}{\sin A} = \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}$

$\Rightarrow (b-c)\cos\frac{A}{2} = a\sin\frac{B-C}{2}$

Q.18 (1)

$a\cos^2 B + a\cos^2 C + \cos A\ \ c\cos C + b\cos B\cos A$

$= \cos B\,(a\cos B + b\cos A) + \cos C\,(a\cos C + c\cos A)$

$= c\cos B + b\cos C = a.$]

Q.19 (3)

$ab^2\cos A + ba^2\cos B + ac^2\cos A + ca^2\cos C$

$+bc^2\cos B + b^2c\cos C$

$= ab(b\cos A + a\cos B) + ac\ c(\cos A + a\cos C)$

$+bc(c\cos B + b\cos C)$

$= abc + abc + abc = 3abc$

Q.20 **(1)**

$c = 5 ; a = 9 ; b = 7$ (Using Napier's Analogy)

$$\frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}} = \frac{a-b}{a+b} = \frac{2}{16} = \frac{1}{8}$$

Q.21 (1)

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2} \Rightarrow$$

$$\tan\left(\frac{90^\circ}{2}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}\cot\frac{A}{2}$$

$$\Rightarrow \tan\left(\frac{A}{2}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{3+1-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$\Rightarrow \frac{A}{2} = 15^o \Rightarrow A = 30^o .$$

Q.22 (1)

We have, $2s = a + b + c, A^2 = s(s-a)(s-b)(s-c)$

$\because$ A.M. $\geq$ G.M .

$$\Rightarrow \frac{s-a+s-b+s-c}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)} \Rightarrow$$

$$\frac{3s-2s}{3} \geq \frac{(A^2)^{1/3}}{s^{1/3}} \Rightarrow \frac{s^3}{27} \geq \frac{A^2}{s} \Rightarrow A \leq \frac{s^2}{3(\sqrt{3})}$$

Q.23 (2)

$s-a = 3 \Rightarrow b+c-a = 6$ ....(i)

$s-c = 2 \Rightarrow a+b-c = 4$ ....(ii)

Adding these two equations, we get $b = 5$

Since *B* is a right angle

$\therefore\ b^2 = a^2 + c^2 \Rightarrow a^2 + c^2 = 25$ ....(iii)

Sol..ving, we get $a = 3, c = 4$

Q.24 (4)

$$\tan\frac{A}{2} \cdot \tan\frac{C}{2} = \frac{5}{6} \cdot \frac{2}{5}$$

$$\sqrt{\frac{(s-b)(s-c)}{s(s-a)} \cdot \frac{(s-b)(s-a)}{s(s-c)}} = \frac{1}{3}$$

$\Rightarrow \dfrac{s-b}{s} = \dfrac{1}{3}$ = 3s – 3b = s = 2s = 3b = a + b + c = 3b

$\Rightarrow 2b = a + c$ $\Rightarrow$ a, b, c are in A.P.]

Q.25 (3)

$\dfrac{1}{\sin^2\frac{A}{2}}, \dfrac{1}{\sin^2\frac{B}{2}}, \dfrac{1}{\sin^2\frac{C}{2}}$ are in A. P.

$$\Rightarrow \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}} = \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}}$$

$$\Rightarrow \frac{ab}{(s-a)(s-b)} - \frac{ac}{(s-a)(s-c)} = \frac{ac}{(s-a)(s-c)} - \frac{bc}{(s-b)(s-c)}$$

$$\Rightarrow \left(\frac{a}{s-a}\right)\left(\frac{b(s-c)-c(s-b)}{(s-b)(s-c)}\right) = \left(\frac{c}{s-c}\right)\left(\frac{a(s-b)-b(s-a)}{(s-a)(s-b)}\right)$$

$\Rightarrow abs - abc - acs + abc = acs - abc - bcs + abc$

$\Rightarrow ab - ac = ac - bc \Rightarrow ab + bc = 2ac$

or $\dfrac{1}{c} + \dfrac{1}{a} = \dfrac{2}{b}$, *i.e.*, a, b, c are in H. P.

Q.26 (2)

$$\frac{\tan\frac{A}{2} - \tan\frac{B}{2}}{\tan\frac{A}{2} + \tan\frac{B}{2}} = \frac{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} - \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}$$

$$= \frac{(s-b)\sqrt{s(s-c)} - (s-a)\sqrt{s(s-c)}}{(s-b)\sqrt{s(s-c)} + (s-a)\sqrt{s(s-c)}}$$

$$= \frac{\sqrt{s(s-c)}(s-b-s+a)}{\sqrt{s(s-c)}\ (s-b+s-a)} = \frac{a-b}{c}$$

Q.27 (1)

$$a = 3, b = 5, c = 4,\ s = \frac{a+b+c}{2} = \frac{12}{2} = 6$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}} = \sqrt{\frac{2.3}{12}} = \sqrt{\frac{1}{2}}$$

$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}} = \sqrt{\frac{6.1}{12}} = \sqrt{\frac{1}{2}}$$

$$\therefore \sin\frac{B}{2} + \cos\frac{B}{2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Q.28 (2)

$$\cot\frac{B}{2}.\cot\frac{C}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}.\frac{s(s-c)}{(s-a)(s-b)}}$$

$=\frac{s}{s-a}$ {Since $3a=b+c$ or $a+b+c=2s=4a$}

$=2a/a=2$

Q.29 (1)

$a=5k, b=6k$ and $c=5k$

$$s=\frac{a+b+c}{2}=\frac{5k+6k+5k}{2}=8k$$

$$r=\frac{\Delta}{s}=\sqrt{\frac{s(s-a)(s-b)(s-c)}{s}}$$

$$r=\sqrt{\frac{8k(8k-5k)(8k-6k)(8k-5k)}{8k}}$$

$$r=\frac{3k}{2}\Rightarrow k=\frac{2r}{3}=\frac{2\times 6}{3}=4$$

Q.30 **B**

$$r=\frac{\Delta}{s}, r_1=\frac{\Delta}{s-a}, r_2=\frac{\Delta}{s-b}, r_3=\frac{\Delta}{s-c}$$

$$\Rightarrow r.r_1.r_2.r_3=\frac{\Delta}{s}.\frac{\Delta}{(s-a)}.\frac{\Delta}{(s-b)}.\frac{\Delta}{(s-c)}$$

$\Rightarrow r.r_1.r_2.r_3=\Delta^2$ $\{\Delta^2=s(s-a)(s-b)(s-c)\}$

Q.31 **A**

If $r_1>r_2>r_3\Rightarrow\frac{\Delta}{s-a}>\frac{\Delta}{s-b}>\frac{\Delta}{s-c}$

$\Rightarrow (s-a)<(s-b)<(s-c)$

$\Rightarrow -a<-b<-c$

$\Rightarrow a>b>c$

Q.32 (2)

$$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$\Rightarrow r=4R\sin^3 30^o$, $\{\because A=B=C=60^o\}$

$$\Rightarrow r=\frac{R}{2}$$

Q.33 (2)

$R=\frac{abc}{4\Delta}$, where $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$$a=13, b=12, c=5, s=\frac{30}{2}=15$$

$$\Delta=\sqrt{15(2)(3)10}=3\times 2\times 5=30$$

$$\therefore R=\frac{13\times 12\times 5}{4\times 30}=\frac{13}{2}$$

Q.34 (3)

$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{1}{2R}\Rightarrow\sin A=\frac{a}{2R}$ ETC.

Therefore

$$2R^2\sin A\sin B\sin C=2R^2\frac{a}{2R}.\frac{b}{2R}.\frac{c}{2R}=\frac{abc}{4R}=\Delta$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 **(3)**

$\because$ $A:B:C=3:5:4$

$\therefore$ $A=45°, B=75°, C=60°$

$\because$ from Sine - rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$\Rightarrow\frac{a}{\frac{1}{\sqrt{2}}}=\frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}}=\frac{c}{\frac{\sqrt{3}}{2}}=k$ $(\because \sin 75°=\sin(45°+30°))$

$\therefore$ $a=\frac{k}{\sqrt{2}}$, $b=\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k$ and $c=\frac{k\sqrt{3}}{2}$

$$\therefore\ a+b+c\sqrt{2}=\frac{k}{\sqrt{2}}+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k+\left(\frac{k\sqrt{3}}{2}\right)\sqrt{2}$$

$$=\frac{k}{2\sqrt{2}}\left[2+(\sqrt{3}+1)+2\sqrt{3}\right]=\frac{3k(\sqrt{3}+1)}{2\sqrt{2}}=3b$$

Q.2 **(3)**

given $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$

..... (i)

$\because$ $a=k\sin A, b=k\sin B, c=k\sin C$

$\therefore$ (i) becomes

$$\frac{\cot A}{k}=\frac{\cot B}{k}=\frac{\cot C}{k}$$

$\therefore$ $A=B=C$

$\therefore$ ΔABC is an equilateral triangle

Q.3 **(3)**

$$\because\ \frac{bc\sin^2 A}{\cos A+\cos B\cos C}=$$

$$\frac{k^2\sin B\sin C\sin^2 A}{-\cos(B+C)+\cos B\cos C}$$

$= \dfrac{k^2 \sin B \sin C \sin^2 A}{\sin B \sin C} = k^2 \sin^2 A = a^2.$

Q.4 **(3)**

$(a + b + c)(b + c - a) = k\,bc$

$\Rightarrow (b + c + a)(b + c - a) = kbc$

$\Rightarrow (b + c)^2 - a^2 = kbc$

$\Rightarrow b^2 + c^2 - a^2 + 2bc = kbc$

$\Rightarrow b^2 + c^2 - a^2 = (k - 2)\,bc$

$\Rightarrow \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{k-2}{2}$

$\because -1 < \dfrac{k-2}{2} < 1$

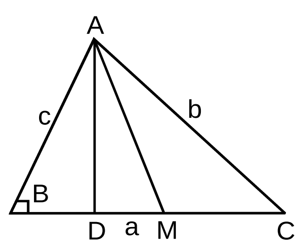
$\Rightarrow -2 < k - 2 < 2 \Rightarrow 0 < k < 4$

Q.5 **(2)**

BM = a/2

BD = c cos B

$DM = \dfrac{a}{2} - c\cos B$



$= \dfrac{a}{2} - c\dfrac{(c^2 + a^2 - b^2)}{2ca}$

$= \dfrac{a^2 - c^2 - a^2 + b^2}{2a} = \dfrac{b^2 - c^2}{2a}$

Q.6 **(4)**

use $\tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$ to get A – B

= 60° and A + B = 150^0 (given)

$\Rightarrow$ A = 105°

Q.7 **(4)**

$a^2 \sin 2C + c^2 \sin 2A = a^2(2\sin C\cos C) + c^2(2\sin A\cos A)$

$= 2a^2\left(\dfrac{2\Delta}{ab}\cos C\right) + 2c^2\left(\dfrac{2\Delta}{bc}\cos A\right)$

$(\because \Delta = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}bc\sin A,\ \therefore \sin C = \dfrac{2\Delta}{ab}, \sin A = \dfrac{2\Delta}{bc})$

$= 4\Delta\left\{\dfrac{a\cos C + c\cos A}{b}\right\} = 4\Delta\left(\dfrac{b}{b}\right) = 4\Delta$

Q.8 **(2)**

Given In a ΔABC, $\sin A = \dfrac{2\Delta}{bc}$

$\Rightarrow \sin A = \dfrac{2[a^2 - b^2 - c^2 + 2bc]}{bc}$

$\Rightarrow \sin A = 2[2 - 2\cos A] \Rightarrow \sin A = 4[1 - \cos A]$

$\Rightarrow 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = 4 \,.\, 2\sin^2\dfrac{A}{2}$

$\Rightarrow \tan\dfrac{A}{2} = \dfrac{1}{4}$

$\Rightarrow \tan A = \dfrac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}} = \dfrac{2 \times \frac{1}{4}}{1 - \left(\frac{1}{4}\right)^2} = \dfrac{\frac{1}{2}}{\frac{15}{16}} = \dfrac{1}{2} \times \dfrac{16}{15}$

$\Rightarrow \tan A = \dfrac{8}{15}$

Q.9 **(2)**

$A.M. = \dfrac{a+b+c}{3}$

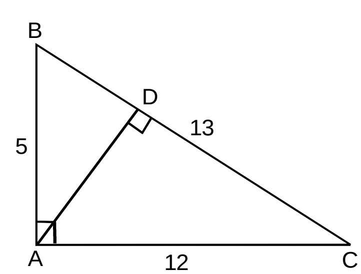
Altitudes are $\dfrac{2\Delta}{a}, \dfrac{2\Delta}{b}, \dfrac{2\Delta}{c}$

$H.M. = \dfrac{3}{\frac{a}{2\Delta} + \frac{b}{2\Delta} + \frac{c}{2\Delta}} = \dfrac{6\Delta}{(a+b+c)}$

$(A.M.) \times (H.M.) = \dfrac{(a+b+c)}{3} \times \dfrac{3 \times 2\Delta}{(a+b+c)} = 2\Delta$

Q.10 **(2)**

Given c = 5, b = 12, a = 13



$\cos A = \dfrac{12^2 + 5^2 - 13^2}{2(12)(5)} = 0$

i.e. A = 90°

Altitude AD = $\frac{2\Delta}{a} = \frac{2\times 30}{13} = \frac{60}{13}$

$\{\Delta = \frac{1}{2}\times 5\times 12 = 30\}$

Q.11 (2)

$\because$ s – a = 3 ...(1)

and s – c = 2 ...(2)

by (1) – (2), we get

c – a = 1

(1) + (2), we get 2s – a – c = 5

$\Rightarrow$ b = 5

$\because$ ΔABC is right angled at B

$\therefore$ $a^2 + c^2 = 25$ ...(3)

$\therefore$ $(c - a)^2 + 2ac = 25$

ac = 12 ...(4)

$\therefore$ $a(1 + a) = 12 \Rightarrow a^2 + a - 12 = 0$

$\Rightarrow (a + 4)(a - 3) = 0$

$\Rightarrow$ a = 3 and c = 4.

Q.12 (1)

$\because\ b\cos^2\frac{A}{2} + a\cos^2\frac{B}{2} = \frac{3}{2}c.$

$\Rightarrow b\,\frac{s(s-a)}{bc} + a\,\frac{s(s-b)}{ac} = \frac{3}{2}c.$

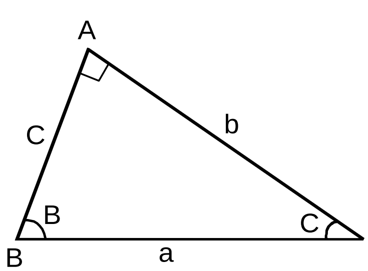
$\Rightarrow \frac{s}{c}\,[s - a + s - b] = \frac{3}{2}c \Rightarrow \frac{s}{c}\times c = \frac{3}{2}c$

$\Rightarrow \frac{a+b+c}{2} = \frac{3c}{2} \qquad \Rightarrow a + b = 2c$

$\Rightarrow$ a, c, b are in A.P.

Q.13 (4)

$\Delta ABC, \angle A = \frac{\pi}{2}$



$\tan\frac{C}{2} = \frac{r}{s-c} = \frac{\Delta}{s(s-c)}\ \{\because \Delta = \frac{1}{2}bc\}$

$\tan\frac{C}{2} = \frac{\frac{1}{2}bc}{\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}} = \frac{2bc}{2b^2 + 2ab}$

$\{\because a^2 = b^2 + c^2\}$

$= \frac{2bc}{2b(b+a)} = \frac{c}{a+b}\times\frac{a-b}{a-b}$

$= \frac{c(a-b)}{a^2 - b^2} = \frac{c(a-b)}{c^2} = \frac{a-b}{c}$

Q.14 (2)

$\because\ A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ and Area = $\frac{9\sqrt{3}}{2}$ cm^2

$\because\ \Delta = \frac{1}{2}bc\sin A \qquad \Rightarrow \frac{9\sqrt{3}}{2} = \frac{1}{2}bc\sin\frac{2\pi}{3} \Rightarrow bc = 18$

$\because\ \cos\frac{2\pi}{3} = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{1}{2}$

$\Rightarrow \frac{(b-c)^2 + 2bc - a^2}{2bc} = -\frac{1}{2}$

$\Rightarrow$ a = 9

Q.15 (2)

$\because\ \frac{b^2 - c^2}{2aR} = \frac{4R^2\left(\sin^2 B - \sin^2 C\right)}{2.2R\sin A.R}$

$= \frac{\sin(B+C).\sin(B-C)}{\sin A} = \sin(B - C)$

Q.16 (2)

$\cos\frac{\pi}{6} = \frac{b^2 + c^2 - a^2}{2bc} = \frac{\sqrt{3}}{2} \Rightarrow a = 1$

$2R = \frac{a}{\sin A} \Rightarrow R = 1$

Q.17 (1)

$\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$

$= \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2R(\sin A + \sin B + \sin C)}$

$= \frac{4\sin A\sin B\sin C}{8\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \qquad = \frac{r}{R}.$

Q.18 (2)

Given a : b : c = 3 : 7 : 8

We know $R = \frac{a}{2\sin A}$ & $r = \frac{\Delta}{s}$

$\Rightarrow \frac{R}{r} = \frac{as}{2\Delta \sin A} = \frac{a(a+b+c)}{4\times\frac{1}{2}bc\sin^2 A}$

$= \frac{a(a+b+c)}{2bc\sin^2 A}$ $\{\Delta = \frac{1}{2}$ bc sin A$\}$ $\{\because$ a = 3k, b= 7k, c = 8k$\}$

$\because \cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{k^2}{k^2}\ \frac{49+64-9}{2.7.8} = \frac{104}{2.7.8} = \frac{13}{14}$

$\therefore \sin^2 A = 1 - \frac{13^2}{14^2} = \frac{27}{14^2}$

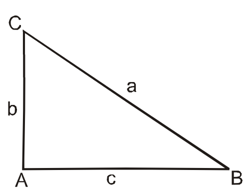
$\Rightarrow \frac{R}{r} = \frac{3(3+7+8)}{2\times7\times8\sin^2 A}\ \frac{k^2}{k^2} = \frac{3\times18\times14\times14}{2\times7\times8\times27} \Rightarrow$

$\frac{R}{r} = \frac{14}{4} = \frac{7}{2} \Rightarrow R : r = 7 : 2$

Q.19 **(2)**

$\because R = \frac{a}{2}$

$\because r = (s-a)\tan\frac{A}{2}$



$\Rightarrow r = (s-a) \Rightarrow r = s - 2R \Rightarrow R = \frac{s-r}{2}$

Q.20 **(3)**

Given $\Delta = 100$ sq. cm, $r_1 = 10$ cm, $r_2 = 50$ cm

$r_1 = \frac{\Delta}{s-a} \Rightarrow s-a = \frac{100}{10} \Rightarrow s-a = 10$ ....(1)

$r_2 = \frac{\Delta}{s-b} \Rightarrow s-b = \frac{100}{50} \Rightarrow s-b = 2$ ....(2)

(1) - (2) $\Rightarrow b - a = 10 - 2 = 8$

Q.21 **(1)**

Given $\angle B = \frac{\pi}{2}$

$r = (s-b)\tan\frac{B}{2} = (s-b)\tan\frac{\pi}{4}$

$= \frac{a+b+c}{2} - b = \frac{c+a-b}{2} = \frac{AB+BC-CA}{2}$

Q.22 **(1)**

H is orthocentre of ΔABC

Radius of circumcircle of ABC,

BHC, CHA, AHB are same and equal to R

$\angle BHC = B + C$

In ΔBHC, by sin law

$\Rightarrow \frac{a}{\sin(B+C)} = 2R$ & $\frac{a}{\sin A} = 2R$

$\Rightarrow \sin(B+C) = \sin A \Rightarrow R_1 = R$

similarly $R_2 = R$ & $R_3 = R$

Q.23 **(1)**

$\frac{a-b}{b-c} = \frac{s-a}{s-c} \Rightarrow \frac{(s-b)-(s-a)}{(s-c)-(s-b)} = \frac{(s-a)}{(s-c)}$

$\Rightarrow (s-b)(s-c) - (s-a)(s-c)$

$= (s-a)(s-c) - (s-a)(s-b)$

$\Rightarrow (s-b)(s-c) + (s-a)(s-b) = 2(s-a)(s-c)$

divided by $(s-a)(s-b)(s-c)$

$\Rightarrow \frac{1}{(s-a)} + \frac{1}{(s-c)} = \frac{2}{(s-b)}$

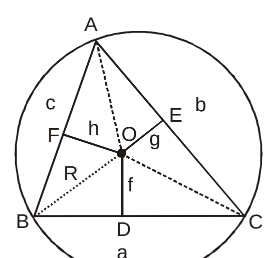
$\Rightarrow \frac{\Delta}{(s-a)} + \frac{\Delta}{(s-c)} = \frac{2\Delta}{(s-b)}$

$\Rightarrow r_1 + r_3 = 2r_2 \Rightarrow r_1, r_2, r_3$ in A.P.

Q.24 **(1)**

In ΔBOD,

$\tan B = \frac{\frac{a}{2}}{f} = \frac{a}{2f}$



$\Sigma \tan A = \pi \tan A$

$\Rightarrow \frac{a}{2f} + \frac{b}{2g} + \frac{c}{2h} = \frac{abc}{8fgh}$

$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{abc}{4fgh}$

$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \frac{abc}{fgh} \quad \therefore \lambda = \frac{1}{4}$

Q.25 (1)

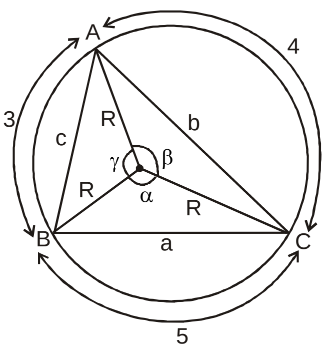
$2\pi R = 3 + 4 + 5 \Rightarrow R = \frac{6}{\pi}$

$\alpha = \frac{3 \times 360}{3 + 4 + 5} = \frac{3 \times 360}{12}$

$\alpha = 90,\ \beta = 120,\ \gamma = 150$

$\Delta ABC = \Delta OBC + \Delta OCA + \Delta OAB$

$\Delta ABC = \frac{1}{2} R.R \sin\alpha + \frac{1}{2} R.R \sin\beta + \frac{1}{2} R.R\sin\gamma$



$= \frac{1}{2} R^2[\sin 90^o + \sin 120^o + \sin 150^o]$

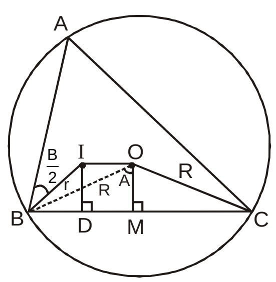
$= \frac{R^2}{2}\left[1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \frac{1}{2}\left(\frac{6}{\pi}\right)^2 \left[\frac{3+\sqrt{3}}{2}\right]$

$= \frac{36}{4\pi^2}(3 + \sqrt{3}) = \frac{9\sqrt{3}}{\pi^2}(1 + \sqrt{3})$

Q.26 (2)

In ΔOBM, $\cos A = \frac{r}{R}$

$\Rightarrow \cos A + \cos B + \cos C = 1 + 4\pi \sin\frac{A}{2}$



$\Rightarrow \frac{r}{R} + \cos B + \cos C = 1 + \frac{r}{R}$

$\Rightarrow \cos B + \cos C = 1$

Q.27 (3)

$AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

$\Rightarrow 4AD^2 = 2b^2 + 2c^2 - a^2$

$\Rightarrow 4BE^2 = 2c^2 + 2a^2 - b^2$

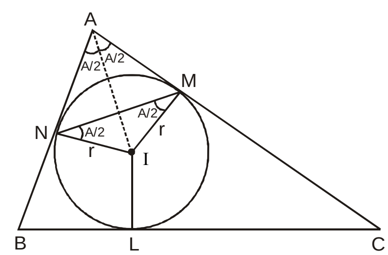
$\Rightarrow 4CF^2 = 2a^2 + 2b^2 - c^2$

$\therefore 4(AD^2 + BE^2 + CF^2) = 3(a^2 + b^2 + c^2)$

$\Rightarrow \frac{AD^2 + DE^2 + CF^2}{BC^2 + CA^2 + AB^2} = \frac{3}{4} = 3:4$

Q.28 (3)

r is in radius



Circumradius is x

$x = \frac{r}{2\sin\frac{A}{2}}$ |||'y, $y = \frac{r}{2\sin\frac{B}{2}}$ & $z = \frac{r}{2\sin\frac{C}{2}}$

$\Rightarrow xyz = \frac{r^3}{8\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)} = \frac{r^3}{8\frac{r}{4R}} = \frac{1}{2} r^2 R$

Q.29 (2)

$r = \frac{\Delta}{s},\ r_1 = \frac{\Delta}{s-a},\ s = \frac{a+b+c}{2}$

$\frac{r}{r_1} = \frac{s-a}{s} = \frac{1}{2} \Rightarrow s = 2a$

$\because \tan\frac{A}{2} = \frac{r}{s-a}$

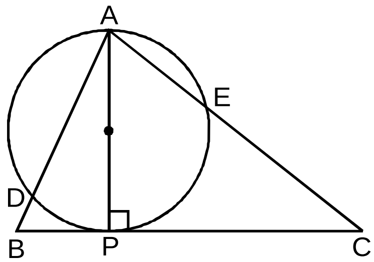
$\tan\frac{A}{2}\left[\tan\frac{B}{2} + \tan\frac{C}{2}\right] = \frac{r}{s-a}\left[\frac{r}{s-b} + \frac{r}{s-c}\right]$

$= \frac{r^2(2s-b-c)}{(s-a)(s-b)(s-c)} = \frac{r^2(a+b+c-b-c)}{\left(\frac{\Delta^2}{s}\right)}$

$= \frac{\Delta^2}{s^2}\ \frac{s}{\Delta^2} \times a = \frac{a}{s} = \frac{a}{2a} = \frac{1}{2}$

Q.30 **(4)**

Sine Rule in ΔADE



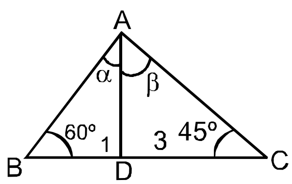
$$\frac{DE}{\sin A} = AP$$

$$\Rightarrow DE = \frac{2\Delta}{a}\sin A = 2\Delta \times \frac{1}{R} = \frac{\Delta}{R}\left\{\frac{\sin A}{a} = \frac{1}{2R}\right.$$

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (C)



if we apply Sine-Rule in Δ BAD, we get

$$\frac{BD}{\sin\alpha} = \frac{AD}{\text{Sin}\,60°}$$

...(1)

if we apply Sine-Rule in Δ CAD, we get.

$$\frac{CD}{\sin\beta} = \frac{AD}{\sin 45°}$$

...(2)

divide (2) by (1)

$$\frac{\sin\alpha}{\sin\beta} \times \frac{CD}{BD} = \frac{\sin 60°}{\sin 45°}$$

$$\frac{\sin\alpha}{\sin\beta} \times \frac{3}{1} = \frac{\sqrt{3}}{2\times\frac{1}{\sqrt{2}}}$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{1}{\sqrt{6}}$$

Q.2 (C)

a = 1

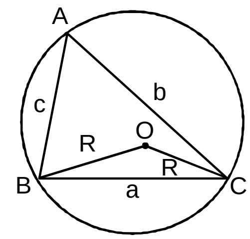
$$(a+b+c) = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$$

$$\Rightarrow (a+b+c) = \frac{2}{2R}(a+b+c) \Rightarrow R = 1$$

$$\because \frac{a}{\sin A} = 2A \Rightarrow \sin A = \frac{1}{2} \Rightarrow A = \frac{\pi}{6}$$

Q.3 (A)

$2(2R)^2 = a^2 + b^2 + c^2$



$$\Rightarrow \left(\frac{a}{2R}\right)^2 + \left(\frac{b}{2R}\right)^2 + \left(\frac{c}{2R}\right)^2 = 2$$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

Q.4 (B)

$$\frac{\cos A}{a} = \frac{\tan C}{c} \Rightarrow \frac{\cos A}{2R\sin A} = \frac{\tan C}{2R\sin C}$$

$\Rightarrow \sin A = \cos A \cos C$

$\Rightarrow \sin(B + C) = \cos A \cos C$

Q.5 (A)

$$\cot A : \cot B : \cot C = 30 : 19 : 6 \begin{cases}\cot A = 30k\\ \cot B = 19k\\ \cot C = 6k\end{cases}$$

$\because \Sigma \cot A \cot B = 1$

$\Rightarrow 570\,k^2 + 114\,k^2 + 180k^2 = 1$

$$\Rightarrow k^2 = \frac{1}{864} \Rightarrow k = \frac{1}{12\sqrt{6}}$$

$$\cot A = \frac{30}{12\sqrt{6}} \Rightarrow \sin A = \frac{2\sqrt{6}}{7} = \frac{10\sqrt{6}}{35}$$

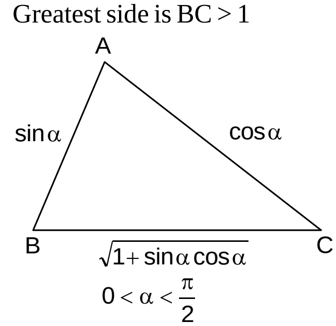
$$\cot B = \frac{19}{12\sqrt{6}} \Rightarrow \sin B = \frac{12\sqrt{6}}{35} = \frac{12\sqrt{6}}{35}$$

$$\cot C = \frac{6}{12\sqrt{6}} \Rightarrow \sin B = \frac{2\sqrt{6}}{5} \Rightarrow \frac{14\sqrt{6}}{35}$$

a : b : c = sinA : sin B : sin C

Q.6 (C)

Greatest side is BC > 1



$$\cos A = \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

$$\cos A = -\frac{1}{2} \Rightarrow A = 120^o$$

Q.7 (C)

$$x^3 - 11x^2 + 38x - 40 = 0 \quad \begin{matrix} a \\ b \\ c \end{matrix}$$

$\Sigma a = 11, \Sigma ab = 38, abc = 40$

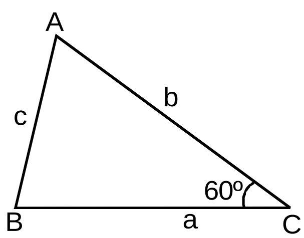
$$\Sigma\frac{\cos A}{a} = \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$

$$= \frac{\frac{1}{2}[b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2]}{abc}$$

$$= \frac{11^2 - 2(38)}{2.(40)} = \frac{45}{2 \times 40} = \frac{9}{16}$$

Q.8 (A)

$$\frac{a\sin B + b\sin A}{\sqrt{\sin A\sin B}} = 4$$



$$\Rightarrow \frac{\frac{ab}{2R} + \frac{ba}{2R}}{\sqrt{\frac{ab}{4R^2}}} = 4$$

$$\Rightarrow \frac{2ab}{2R} \cdot \frac{2R}{\sqrt{ab}} = 4 \qquad \Rightarrow \sqrt{ab} = 2$$

$$a^2 + b^2 + c^2 = 2ab\cos 60^o = 2 \cdot 4 \cdot \frac{1}{2} = 4$$

$= 10 : 12 : 14 \qquad = 5 : 6 : 7$

Q.9 (D)

$\because\ a : b : c = 4 : 5 : 6$

$\therefore\ a = 4k, b = 5k, c = 6k$

$$\because\ \cos B = \frac{c^2 + a^2 - b^2}{2ac} = \frac{9}{16}$$

$$\because\ \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 36 - 16}{2 \times 5 \times 6} = \frac{3}{4}$$

$\because\ \cos 3A = 4\cos^3 A - 3\cos A$

$$= 4 \times \frac{27}{64} - 3 \times \frac{3}{4} = \frac{27}{16} - \frac{9}{4}$$

$$= \frac{27 - 36}{16} = \frac{-9}{16}$$

$\cos 3A = -\cos B = \cos(\pi - B)$

$\therefore\ 3A + B = \pi$

Q.10 (C)

$$\frac{a}{1} = \frac{b}{\sqrt{3}} = \frac{c}{2} = k$$

$$\cos A = \frac{3 + 4 - 1}{4\sqrt{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{6}$$

$$\cos B = \frac{1 + 4 - 3}{2.1.2} = \frac{1}{2} \qquad \Rightarrow B = \frac{\pi}{3} \therefore c = \frac{\pi}{2}$$

$\Rightarrow$ A, B, C in A.P.

Q.11 (A)

$(s - a), (s - b), (s - c)$ in G.P.

$(s - b)^2 = (s - a)(s - c)$

$$\left(\frac{c + a - b}{2}\right)^2 = \left(\frac{b + c - a}{2}\right)\left(\frac{a + b - c}{2}\right)$$

$\Rightarrow a^2 + b^2 + c^2 + 2ac - 2ab - 2bc = b^2 - (a - c)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ac - 2ab - 2bc$

$= b^2 - a^2 - c^2 + 2ac$

$$\Rightarrow \frac{a^2 + c^2}{a + c} = b \Rightarrow \frac{(2R)^2[\sin^2 A + \sin^2 C]}{(2R)[\sin A + \sin C]} = b$$

$$\Rightarrow \frac{\sin^2 A + \sin^2 C}{\sin A + \sin C} = \frac{b}{2R} = \sin B$$

Aliter

$$\frac{s - b}{s - a} = \frac{s - c}{s - b} \text{ Apply C \& D}$$

$$\Rightarrow \frac{2s - a - b}{a - b} = \frac{2s - b - c}{b - c} \Rightarrow \frac{c}{a - b} = \frac{a}{b - c}$$

$\Rightarrow bc - c^2 = a^2 - ab$

$\Rightarrow a^2 + c^2 = b(a + c)$

$\Rightarrow a^2 + c^2 = b(a + c)$

$\Rightarrow (2R)^2(\sin^2 A + \sin^2 C) = (2R)^2 \sin B(\sin A + \sin C)$

$$\Rightarrow \frac{\sin^2 A + \sin^2 C}{\sin A + \sin C} = \sin B$$

Q.12 (B)

$$\cos A = \frac{\sin B}{2\sin C} \Rightarrow 2\sin C\cos A = \sin B$$

$$\Rightarrow 2\frac{c}{2R}\frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{2R} \Rightarrow b^2 + c^2 - a^2 = b^2$$

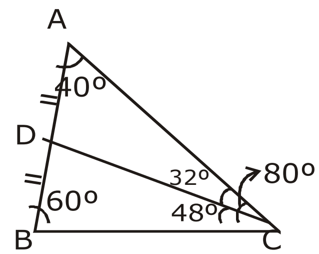
$\Rightarrow c = a \Rightarrow \Delta ABC$ is isosceles

Q.13 (C)

$A + B + C = 180^o$ A, B, C in A.P.

$2B = A + C$ & $C = 2A$

$2B = 3A$



$A + \frac{3A}{2} + 2A = 180^\circ \Rightarrow \frac{9A}{2} = 180^\circ$

$\Rightarrow A = 40^\circ \therefore B = 60^\circ, C = 80^\circ$

$CD = 2\sqrt{3}$ cm

$\frac{\angle BCD}{\angle DCA} = \frac{2}{3} \Rightarrow \angle BCD = 32^\circ; \angle DCA = 48^\circ$

In ΔBCD (using sine rule)

$$\frac{2\sqrt{3}}{\sin 60^\circ} = \frac{\frac{c}{2}}{\sin 32^\circ}$$

$$\Rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{c}{2\sin 32^\circ}$$

$\Rightarrow c = 8\sin 32^\circ$

Q.14 (A)

We have, $\cos(A - B) = \frac{4}{5}$

$$\Rightarrow \frac{1 - \tan^2\left(\frac{A-B}{2}\right)}{1 + \tan^2\left(\frac{A-B}{2}\right)} = \frac{4}{5}$$

$$\Rightarrow \tan^2\left(\frac{A-B}{2}\right) = \frac{1}{9}$$

or $\tan^2\left(\frac{A-B}{2}\right) = \frac{1}{9}$ or $\tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$

(As $a > b \Rightarrow A > B$)

Also, $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

$$\Rightarrow \frac{1}{3} = \left(\frac{6-3}{6+3}\right)\cot\frac{C}{2} \Rightarrow \cot\frac{C}{2} = 1$$

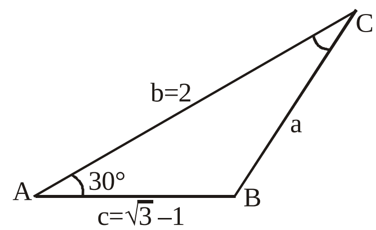
$\therefore C = 90^\circ$

Hence, area of triangle ABC $= \frac{1}{2}ab\sin C = \frac{1}{2} \cdot 6 \cdot 3 \cdot \sin 90^\circ = 9.$

Q.15 (A)

Using $\Delta = \frac{1}{2}bc\sin A$

$$\therefore \quad \frac{1}{2} \cdot 2 \cdot \left(\sqrt{3} - 1\right)\sin A = \frac{\sqrt{3}-1}{2}$$



$\therefore \quad \sin A = \frac{1}{2} \quad \Rightarrow \quad A = 30^\circ$

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

$$= \frac{3-\sqrt{3}}{\sqrt{3}+1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}-1} = \sqrt{3}$$

$\Rightarrow \quad B - C = 120^\circ$

also $\quad B + C = 150^\circ \quad \Rightarrow C = 15^\circ$ **Ans.**]

Q.16 (A)

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\therefore \quad \frac{c+2}{2} \geq \sqrt{\frac{c^2+4}{2}}$$

(AM $\geq$ GM)

$c^2 + 4c + 4 \geq 2c^2 + 8$

$c^2 - 4c + 4 \leq 0$

$(c-2)^2 \leq 0$

$c = 2 \Rightarrow a = b$

$\therefore \quad 2a = 4 \quad \Rightarrow \quad a = 2 = b$

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4} \times a^2 = \sqrt{3}$$

$\Delta^2 = 3.$

Q.17 (C)

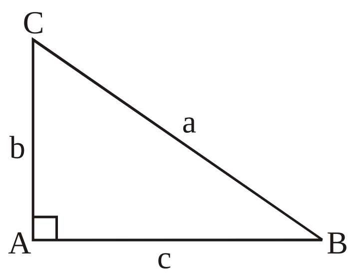
$b\cot B + c\cot C = 2(r + R)$

$$2R\sin B \cdot \frac{\cos B}{\sin B} + 2R\sin C \cdot \frac{\cos C}{\sin C} = 2(r + R)$$

$$\cos B + \cos C = 1 + \frac{r}{R}$$

$\cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

$\cos B + \cos C = \cos A + \cos B + \cos C$



$\therefore \cos A = 0 \Rightarrow A = \frac{\pi}{2}$

$a^2 = b^2 + c^2 \Rightarrow b^2 + c^2 = 100$

Using $AM \geq GM$

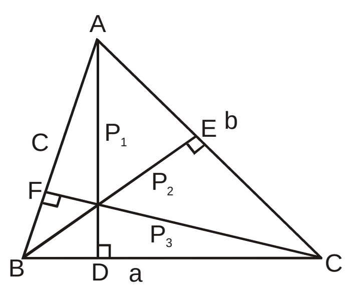
$\frac{b^2 + c^2}{2} \geq \sqrt{b^2c^2} \Rightarrow bc \leq 50$

Hence, area of $\Delta ABC = \frac{1}{2} bc = 25$

Q.18 (C)

$AD = P_1, BE = P_2, CF = P_3$

$\because P_1, P_2, P_3$ in H.P.



$\Rightarrow \frac{1}{P_1}, \frac{1}{P_2}, \frac{1}{P_3}$ in A.P. $\Rightarrow \frac{a}{2\Delta}, \frac{b}{2\Delta}, \frac{c}{2\Delta}$ in A.P.

$\Rightarrow$ a, b, c in A.P. $\Rightarrow$ sinA, sinB, sinC in A.P.

Q.19 (A)

$a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$

$\Rightarrow a \frac{s(s-c)}{ab} + c \frac{s(s-a)}{bc} = \frac{3b}{2}$

$\Rightarrow 2s[2s - c - a] = 3b^2$

$\Rightarrow (a + b + c) = 3b$

$\Rightarrow a + c = 2b$

$\Rightarrow$ a, b, c in A.P.

Q.20 (D)

$1 - \tan \frac{A}{2} \tan \frac{B}{2} = 1 - \frac{\Delta}{s(s-a)} \frac{\Delta}{s(s-b)}$

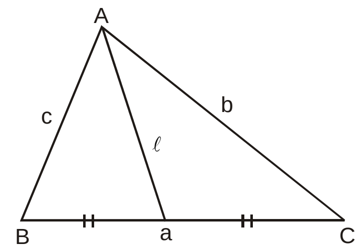
$= 1 - \frac{\Delta^2 (s-c)}{s(s-a)(s-b)(s-c)s}$

$= 1 - \frac{s-c}{s} = \frac{s-s+c}{s} = \frac{c}{s} = \frac{2c}{a+b+c}$

Q.21 (B)

$4l^2 = 2b^2 + 2c^2 - a^2$

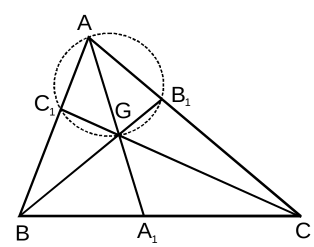
$\Rightarrow 4\ell^2 = 2(b^2 + c^2 - a^2) + a^2$



$\Rightarrow 4\ell^2 = a^2 + 4bc \cos A$

Q.22 (C)

$BC_1 . BA = BG . BB_1$



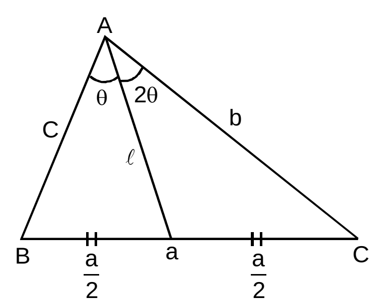
$\frac{c}{2} . c = \frac{2}{3} \ell_B . \ell_B \Rightarrow 3c^2 = 4\ell_B^2$

$\Rightarrow 3c^2 = 4. \frac{1}{4} (2a^2 + 2c^2 - b^2)$

$\Rightarrow c^2 + b^2 = 2a^2$

Q.23 (B)

$\frac{\ell}{\sin B} = \frac{\frac{a}{2}}{\sin \theta}$ & $\frac{\ell}{\sin C} = \frac{\frac{a}{2}}{\sin 2\theta}$



$$\frac{\sin B}{\sin C} = \frac{\sin\theta}{\sin 2\theta} \Rightarrow \frac{\sin B}{\sin C} = \frac{\sin\theta}{2\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\sin B}{\sin C} = \frac{1}{2\cos\theta} \Rightarrow \frac{\sin B}{\sin C} = \frac{1}{2\cos\theta}$$

$$\Rightarrow \frac{\sin B}{\sin C} = \frac{1}{2}\sec\frac{A}{3}$$

Q.24 (B)

$$r = \frac{a}{2}\cot\frac{\pi}{n},\ R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$r + R = \frac{a}{2}\left(\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right) = \frac{a}{2}\left(\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right) = \frac{a}{2}$$

$$\cot\frac{\pi}{2n}$$

Q.25 (B)

$$r = \sqrt{3},\ ID = IE = IF\ \ \&\ r = \frac{\Delta}{s}$$

$$\Rightarrow \Delta = rs = \sqrt{3}\,s$$

$$\text{In } \Delta\, IBD,\ \tan 15^{\circ} = \frac{\sqrt{3}}{\left(b\frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow b = \frac{2}{\tan 15^{\circ}} = 2(2+\sqrt{3})$$

$$\therefore s = \frac{2b+b\sqrt{3}}{2} = \frac{1}{2}(b(2+\sqrt{3}))$$

$$= \frac{2}{2}(2+\sqrt{3})^2 = (7+4\sqrt{3})$$

$$\Delta = \sqrt{3}\,(7+4\sqrt{3}) = 12 + 7\sqrt{3} \text{ sq. units}$$

Q.26 (C)

$\because$ G.M. $\geq$ H.M.

$$(r_1 r_2 r_3)^{1/3} \geq \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} \qquad \Rightarrow (r_1 r_2 r_3)^{1/3} \geq 3r$$

$$\Rightarrow \frac{r_1\ r_2\ r_3}{r^3} \geq 27$$

Q.27 (D)

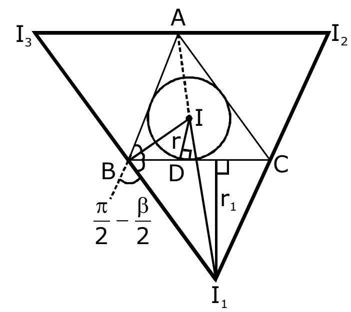
$$\because\ r_1 + r_2 = \frac{\Delta c}{(s-a)(s-b)}$$

$$\therefore\ \Pi\,(r_1 + r_2) = \frac{\Delta^3 abc}{(s-a)^2(s-b)^2(s-c)^2} = \frac{\Delta^3 (abc)s^2}{\Delta^4}$$

$$= \frac{(abc)\ s^2}{\Delta} = \frac{4R\Delta\ s^2}{\Delta} = 4Rs^2$$

$$\therefore\ \frac{\Pi\,(r_1+r_2)}{Rs^2} = 4$$

Q.28 (B)

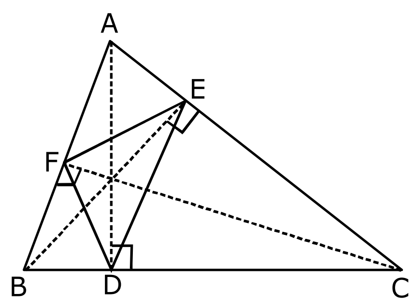


$$\frac{r}{BI} = \sin\frac{B}{2} \Rightarrow BI = r\operatorname{cosec}\frac{B}{2}$$

$$\&\quad \sin\left(\frac{\pi}{2}-\frac{\beta}{2}\right) = \frac{r_1}{BI_1} \Rightarrow BI_1 = \frac{r_1}{\cos\frac{B}{2}}$$

$$II_1 \,.\, II_2 \,.\, II_3 = (4R)^3 \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = 16R^2 r$$

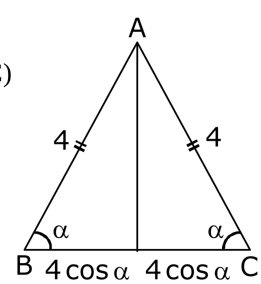
Q.29 (B)



$a' = R\sin 2A, b' = R\sin 2B$
$c' = R\sin 2C,\ B' = \pi - 2B$

$$\Delta = \frac{1}{2}R^2 \sin 2A \sin 2C \,.\, \sin 2B$$

Q.30 (C)



$$Rr = \frac{abc}{4\Delta} \times \frac{\Delta}{s} = \frac{abc}{2(a+b+c)}$$

$$= \frac{4.4.8\cos\alpha}{2(8+8\cos\alpha)} = \frac{8.16\cos\alpha}{16(1+\cos\alpha)} = \frac{8\cos\alpha}{1+\cos\alpha}$$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 A, B

(A) $\because \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$$= \frac{b^2+c^2-a^2}{2abc} + \frac{c^2+a^2-b^2}{2abc} + \frac{a^2+b^2-c^2}{2abc}$$

$$= \frac{a^2+b^2+c^2}{2abc}$$

(B) $\because \frac{\sin A}{a} + \frac{\sin B}{b} + \frac{\sin C}{c}$

$$= \frac{a}{2R.a} + \frac{b}{2R.b} + \frac{c}{2R.c} = \frac{3}{2R}$$

(C) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$\Rightarrow$ cot A = cot B = cot C $\Rightarrow$ A = B = C
true for equilateral triangle only

(D) $\frac{\sin 2A}{a^2} = \frac{\sin 2B}{b^2} = \frac{\sin 2C}{c^2}$

$$\Rightarrow \frac{2\sin A\cos A}{k^2\sin^2 A} = \frac{2\sin B\cos B}{k^2\sin^2 B} = \frac{2\sin C\cos C}{k^2\sin^2 C}$$

$\Rightarrow$ cot A = cot B = cot C

$\Rightarrow$ A = B = C true for equilateral triangle only

Q.2 A, B,C,D

$2a^2 + 4b^2 + c^2 - 4ab - 2ac = 0$

$(a-2b)^2 + (a-c)^2 = 0$

$\Rightarrow$ a = 2b and a = c

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos B = \frac{7}{8}$$

Hence, $\sin(A+C) = \sin(\pi - B) = \sin B$

$$= \sqrt{1-\frac{49}{64}} = \sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{8}$$

Hence, $\frac{8\sqrt{5}}{3}\sin(A+C) = 5.$

Q.3 A, B, C

(A) $\because \tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$ ....(i)

$$\because \tan^2\left(\frac{A-B}{2}\right) = \frac{1-\cos(A-B)}{1+\cos(A-B)} = \frac{1-\frac{31}{32}}{1+\frac{31}{32}} = \frac{1}{63}$$

$\therefore \tan\left(\frac{A-B}{2}\right) = \frac{1}{3\sqrt{7}}$ $\because$ a = 5 and b = 4

$\therefore$ from equation (i), we get

$$\frac{1}{3\sqrt{7}} = \left(\frac{5-4}{5+4}\right)\cot\frac{C}{2} \Rightarrow \frac{1}{3\sqrt{7}} = \frac{1}{9}\cot\frac{C}{2}$$

$$\Rightarrow \cot\frac{C}{2} = \frac{3}{\sqrt{7}}$$

$$\because \cos C = \frac{1-\tan^2 C/2}{1+\tan^2 C/2} = \frac{1-7/9}{1+7/9} = \frac{2}{16} = \frac{1}{8}$$

$$\because \cos C = \frac{b^2+a^2-c^2}{2ab}$$

$\Rightarrow c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c = 6$

(B), (C) $\because$ Area $= \frac{1}{2}ab\sin C$ $\because \cos C = \frac{1}{8} \Rightarrow$

$$\sin C = \sqrt{1-\frac{1}{64}} = \frac{3\sqrt{7}}{8}$$

$$\text{Area} = \frac{1}{2} \times 5 \times 4 \times \frac{3\sqrt{7}}{8}$$

Area $= \frac{15\sqrt{7}}{4}$ sq. unit. $\because$ From Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \sin A = \frac{a\sin C}{c} = \frac{5\times 3\sqrt{7}}{6\times 8}$$

$$\therefore \sin A = \frac{5\sqrt{7}}{16}$$

Q.4 A, C, D

$\because \beta_a = \frac{2bc}{b+c}\cos\frac{A}{2}$

(A) correct

(B) incorrect

(C) $\frac{abc\,\text{cosec}\frac{A}{2}}{2R(b+c)} = \frac{abc\,\text{cosec}\frac{A}{2}}{\frac{a}{\sin A}.(b+c)}$

$$= \frac{bc\ 2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}.(b+c)} = \frac{2bc}{(b+c)}\cos\frac{A}{2}$$

(D) $\because \frac{2\Delta}{(b+c)}\text{cosec}\frac{A}{2} = \frac{bc\sin A}{(b+c)}.\frac{1}{\sin\frac{A}{2}}$

$$= \frac{2bc\sin\frac{A}{2}\cos\frac{A}{2}}{(b+c)}.\frac{1}{\sin\frac{A}{2}} = \frac{2bc}{b+c}\cos\frac{A}{2}$$

Q.5 **A, C**

$$\frac{AI}{ID} = \frac{b+c}{a} = \frac{2}{1};$$

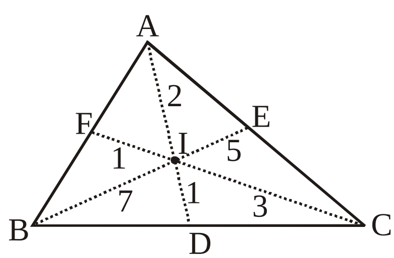
$$\frac{BI}{IE} = \frac{a+c}{b} = \frac{7}{5};$$

$$\frac{CI}{IF} = \frac{a+b}{c} = \frac{3}{1}$$

$$\frac{b+c-a}{b+c+a} = \frac{1}{3};\ \frac{a+c-b}{a+b+c} = \frac{1}{6};\ \frac{a+b-c}{a+b+c} = \frac{1}{2}$$

$$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$$

$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s^3}} = \frac{1}{6}$$



$$\Rightarrow r + \Delta = \frac{\Delta}{s} + \Delta = \Delta\left(\frac{1}{s}+1\right) = \frac{s^2}{6}\left(1+\frac{1}{5}\right)$$

$$= \frac{24\times 13}{12} = 26$$

Q.6 **B, C, D**

$r = r_2 + r_3 - r_1$

$$\frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s-a}$$

$$\frac{(s-a)(s-c)}{s(s-b)} = \frac{a-c}{b}$$

$$\tan^2\left(\frac{B}{2}\right) = \left(\frac{a-c}{b}\right)$$

Since $\frac{B}{2} \in \left(\frac{\pi}{6}, \frac{\pi}{4}\right)$

$\therefore \quad \frac{a-c}{b} \in \left(\frac{1}{3}, 1\right).$

Q.7 **B,D**

$r_1 = 2r_2 = 3r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$$

$$\Rightarrow \underset{(i)}{\frac{1}{s-a}} = \underset{(ii)}{\frac{2}{s-b}} = \underset{(iii)}{\frac{3}{s-c}}$$

From (i) and (ii) we get $a - b = c/3$ ...(1)
From (i) and (iii), we get $2a - b = 2c$ ...(2)
From (ii) and (iii), we get $a - 5b = -5c$ ...(3)
let $c = k$, then from (1) and (2), we get

$a = \frac{5k}{3}$ and $b = \frac{4k}{3}$

$\therefore \frac{a}{b} = \frac{5}{4};\ \frac{a}{c} = \frac{5}{3}.$

Q.8 **A, B, C, D**

(A) $\frac{\Delta}{s-b} + \frac{\Delta}{s-c} = \frac{\Delta}{s-a} - \frac{\Delta}{s}$

$$\Rightarrow \frac{1}{(s-b)(s-c)} = \frac{1}{s(s-a)}$$

$\Rightarrow \tan^2\frac{A}{2} = 1 \Rightarrow A = 90°$

(B) $4R^2(\sin^2A + \sin^2B + \sin^2C) = 8R^2$
$1 - (\cos^2A - \sin^2B) + \sin^2C = 2$
$1 - \cos(A + B)\cos(A - B) + 1 - \cos^2C = 2.$
$\cos C\cos(A - B) - \cos^2C = 0$
$\cos C[\cos(A - B) - \cos C] = 0$
$\cos C[\cos(A - B) + \cos(A + B)] = 0$
$2\cos A\cos B\cos C = 0$
$\Rightarrow A = 90°$ or $B = 90°$ or $C = 90°$

(C) $r_1 = s.$
$s\tan A/2 = s \Rightarrow \tan A/2 = 1 \Rightarrow A = 90°$

(D) $\frac{a}{\sin A} = \frac{a\Delta}{s(s-a)} \Rightarrow \frac{1}{\sin A} = \tan A/2$

$\Rightarrow\ 2\sin^2 A/2 = 1$

$\Rightarrow\ 1 - \cos A = 1 \Rightarrow \cos A = 0 \Rightarrow A = 90°$

Q.9 **A, B**

$\because\ \sin C = \dfrac{1-\cos A\cos B}{\sin A\sin B} \le 1$

$\Rightarrow \cos(A-B) \ge 1$

$\Rightarrow \cos(A-B) = 1$

$\Rightarrow A - B = 0 \qquad \Rightarrow A = B$

$\therefore\ \sin C = \dfrac{1-\cos^2 A}{\sin^2 A} = 1$

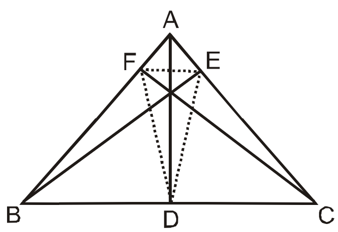
$\Rightarrow C = 90°$

Q.10 **A, B. C. D**

$\because\ FE = a\cos A = R\sin 2A$

$DE = c\cos C = R\sin 2C$

$FD = b\cos B = R\sin 2B$



(A) $= \dfrac{R\left(\sum \sin 2A\right)}{a+b+c}$

$= \dfrac{R(4\sin A\sin B\sin C)}{2R\left(4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)}$

$= \dfrac{8\left(\prod \sin\frac{A}{2}\right)\left(\prod \cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \dfrac{r}{R}$

(B) $\because$ Area of ΔDEF

$= \dfrac{1}{2} FD \times DE \sin(\pi - 2A)$

$= \dfrac{1}{2} b\cos B.c\cos C.\sin 2A$

$= \dfrac{1}{2} bc\cos B.\cos C.2\sin A.\cos A$

$= 2\left(\dfrac{1}{2}bc\sin A\right)\cos A.\cos B.\cos C$

$= 2\Delta\cos A.\cos B.\cos C$

(C) Area of $\Delta AEF = \dfrac{1}{2} AE \times AF \sin A$

$= \dfrac{1}{2}(c\cos A)(b\cos A)\sin A$

$= \left(\dfrac{1}{2}bc\sin A\right)\cos^2 A$

$= \Delta\cos^2 A$

(D) $R_{DEF} = \dfrac{FE \times DE \times FD}{4\Delta_{DEF}}$

$= \dfrac{abc\cos A\cos B\cos C}{4\times 2\Delta\cos A\cos B\cos C}$

$= \dfrac{abc}{8\Delta} = \dfrac{4R\Delta}{8\Delta} = \dfrac{R}{2}$

Q.11 **B,D**

Product of distances of incenter from angular points

$= \dfrac{r^3}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \dfrac{r^3}{r/4R}$

$= 4Rr^2$

$= \dfrac{abc}{\Delta}r^2 = \dfrac{(abc)(r)}{\frac{\Delta}{r}}$

$= \dfrac{(abc)(r)}{s}$.

Q.12 **A,B**

(A) $\dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c}$

$=$

$\dfrac{a^2+b^2-c^2+a^2+c^2-b^2+b^2+c^2-a^2}{abc}$

$= \dfrac{a^2+b^2+c^2}{2abc}$

(B) $\dfrac{\sin A}{a} + \dfrac{\sin B}{b} + \dfrac{\sin C}{c} = \dfrac{1}{2R} + \dfrac{1}{2R} + \dfrac{1}{2R} = \dfrac{3}{2R}$

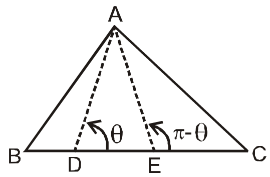
(C) $\dfrac{\sin 2A}{a^2} = \dfrac{\sin 2B}{b^2} = \dfrac{\sin 2C}{c^2}$

$\Rightarrow \dfrac{2\cos A}{2Ra} = \dfrac{2\cos B}{2Rb} = \dfrac{2\cos C}{2Rc}$

$\Rightarrow \dfrac{\cos A}{a} = \dfrac{\cos B}{b} = \dfrac{\cos C}{c}$

$\Rightarrow \Delta ABC$ equilateral

Q.13 **A, C, D**



if we apply m-n Rule in Δ ABE, we get
$(1+1)\cot\theta = 1.\cot B - 1.\cot\theta$
$2\cot\theta = \cot B - \cot\theta$
$3\cot\theta = \cot B$
$\tan\theta = 3\tan B$
..........(1)
Similarly, if we apply m-n Rule in Δ ACD, we get
$(1+1)\cot(\pi-\theta) = 1.\cot\theta - 1.\cot C$.
$\cot C = 3\cot\theta$ $\Rightarrow \tan\theta = 3\tan C$
..........(2)
form (1) and (2) we can say that
$\tan B = \tan C \Rightarrow B = C$
$\because A + B + C = \pi$
$\therefore A = \pi - (B + C)$
$= \pi - 2B \because B = C$
$\therefore \tan A = -\tan 2B$

$$= -\left(\frac{2\tan B}{1-\tan^2 B}\right) = -\frac{\frac{2\tan\theta}{3}}{1-\frac{\tan^2\theta}{9}}$$

$$\Rightarrow \tan A = \frac{6\tan\theta}{\tan^2\theta - 9}$$

Q.14 **A, C, D**

$$\because r_1 - r = \frac{\Delta}{s-a} - \frac{\Delta}{s} = \frac{\Delta a}{s(s-a)} = a\tan\frac{A}{2}$$

$$\therefore \Pi(r_1 - r) = abc\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$$

$$= abc\,\Pi\tan\frac{A}{2}$$

$$= abc\,\frac{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$= \frac{(abc)r}{4R.\frac{(\sin A+\sin B+\sin C)}{4}} = \frac{(abc)r}{R\left(\frac{a}{2R}+\frac{b}{2R}+\frac{c}{2R}\right)}$$

$$= \frac{2(abc)r}{2s} = \frac{4R\Delta r}{s} = 4Rr^2$$

Comprehension # 1 (Q. no. 15 to 17)

Q.15 (C)

$$\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = b\sin B + c\sin C + a\sin A = \frac{b^2+c^2+a^2}{2R}$$

$\therefore$ $k = 2R$

Q.16 **(C)**

$$\because \cot A + \cot B + \cot C = \frac{R}{abc}(b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2)$$

$$= \frac{R}{abc}(b^2 + c^2 + a^2) = \frac{R}{abc}\left(\frac{4\Delta^2}{x^2}+\frac{4\Delta^2}{y^2}+\frac{4\Delta^2}{z^2}\right)$$

$$= \frac{4\Delta^2 R}{abc}\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$$

$$= \frac{4\Delta R}{abc}.\Delta\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right) = \Delta\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$$

$\therefore$ $k = \Delta$

Q.17 **D**

$$\sum\frac{c\sin B + b\sin C}{x} = \sum\frac{x+x}{x} = 6$$

Comprehension # 2 (Q. no.18 to 20)

Q.18 **A**
Q.19 **D**
Q.20 **B**

$$\text{Given } \log\left(\frac{a+c}{a}\right) + \log\left(\frac{a}{b}\right) = \log 2$$

$$\Rightarrow \log\left(\frac{a+c}{b}\right) = \log 2 \Rightarrow a+c=2b \;...(1)$$

$\Rightarrow$ A.P.
Also $a - ax^2 + 2bx + c + cx^2 = 0$
$(c-a)x^2 + 2bx + (c+a) = 0$ has equal roots
$\therefore$ $D = 0 \Rightarrow 4b^2 - 4(c^2 - a^2) = 0$
$\therefore$ $b^2 = c^2 - a^2$
....(2)
$\Rightarrow \Delta ABC$ is right angled at C $\Rightarrow \angle C = 90°$
From (1) and (2), $b^2 = (c-a)(c+a)$
$\Rightarrow b^2 = (c-a)\,2b \Rightarrow 2(c-a) = b$
...(3)
As $\angle C = 90° \Rightarrow A + B = 90°$
From (3) using Sine Law
$2(\sin C - \sin A) = \sin B$
$C = 90° \Rightarrow \sin C = 1 ; A + B = 90°$
$\Rightarrow B = 90° - A$
$2(1 - \sin A) = \sin(90° - A) = \cos A$

squaring both sides

$\Rightarrow$ $4(1 - \sin A)^2 = \cos^2 A = (1 - \sin^2 A)$

$\Rightarrow$ $4(1 - \sin A) = (1 + \sin A)$

$\Rightarrow$ $3 = 5 \sin A \Rightarrow \sin A = 3/5$

$B = 90 - A$

$\sin B = \cos A = 4/5$ and $\sin C = 1$

$\Rightarrow \sin A + \sin B + \sin C = \frac{3}{5} + \frac{4}{5} + 1 = \frac{12}{5}$

Comprehension # 3 (Q. no. 21 to 23)

Q.21 **D**

$\because PG = \frac{1}{3} AD$

$= \frac{1}{3}.\frac{2\Delta}{a}$

$= \frac{2}{3a}.\frac{1}{2}.ab \sin C$

or $= \frac{1}{3} b \sin C \; (\because \Delta = \frac{1}{2} ac \sin B)$

$\therefore PG = \frac{2}{3a}.\frac{1}{2} ac \sin B$

$= \frac{1}{3} c \sin B$

Q.22 **B**

$\because$ Area of $\Delta GPL = \frac{1}{2}(PL)(PG)$

and Area of $\Delta ALD = \frac{1}{2}(DL)(AD) \because PL = \frac{1}{3} DL$

and $PG = \frac{1}{3} AD$

$$\therefore \frac{\text{Area of } \Delta GPL}{\text{Area of } \Delta ALD} = \frac{\frac{1}{2}(PL)(PG)}{\frac{1}{2}(DL)(AD)}$$

$$= \frac{\frac{1}{3}(DL)\times\frac{1}{3}(AD)}{(DL)(AD)} = \frac{1}{9}$$

Q.23 **B**

$\because$ Area of ΔPQR = Area of ΔPGQ + Area of ΔQGR + Area of ΔRGP ...(1)

$\because$ Area of $\Delta PGQ = \frac{1}{2} PG.GQ.\sin(\angle PGQ)$

$= \frac{1}{2} \times \frac{1}{3} AD \times \frac{1}{3} BE \sin(\pi - C)$

$= \frac{1}{18} \times \frac{2\Delta}{a} \times \frac{2\Delta}{b} \sin C$

$= \frac{2}{9ab} \times \frac{1}{2} bc \sin A \times \frac{1}{2} ac \sin B \times \sin C$

$= \frac{c^2}{18} \sin A.\sin B.\sin C$

Similarly Area of $\Delta QGR = \frac{a^2}{18} \sin A.\sin B.\sin C$ and

Area of $\Delta RGP = \frac{b^2}{18} \sin A.\sin B.\sin C$

$\therefore$ From equation (1), we get

Area of $\Delta PQR = \frac{1}{18}(a^2 + b^2 + c^2) \sin A.\sin B.\sin C$

Q.24 (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (r)

(A) $\because 2B = A + C \Rightarrow B = \frac{\pi}{3}$ and $A + C = \frac{2\pi}{3}$

$\because b^2 = ac$

$\Rightarrow \sin^2 B = \sin A.\sin C$

$\Rightarrow \sin A \sin C = \frac{3}{4}$

$\Rightarrow \cos(A - C) - \cos(A + C) = \frac{3}{2} \because A + C = \frac{2\pi}{3}$

$\Rightarrow \cos(A - C) = 1$

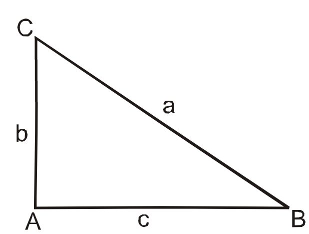
$\Rightarrow A = C = \frac{\pi}{3} = B$

$\Rightarrow a = b = c$

$\therefore \frac{a^2(a+b+c)}{3abc} = 1$

(B) $\because a^2 = b^2 + c^2$ and $2R = a$

$\therefore \frac{a^2 + b^2 + c^2}{R^2} = \frac{2a^2}{R^2} = 8$



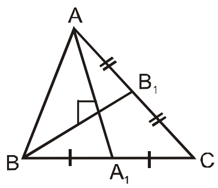
(C) $\because \Delta = \frac{1}{2} bc \sin A$

$\Rightarrow \Delta = \frac{1}{2}.9.\sin A = \frac{9}{2} \times \frac{a}{2R} \because a = 2$

$\therefore 2R\Delta = 9$

(D) $\because$ a = 5, b = 3 and c = 7
and because we know that
b cos C + c cos B = a
$\therefore$ 3 cos C + 7 cos B = 5

Q.25 (A) $\to$ (s),(B) $\to$ (p), (C) $\to$ (r),(D) $\to$ (q)



Match the column
(A) $\because$ AA_1 and BB_1 are perpendicular
$\therefore$ $a^2 + b^2 = 5c^2$

$\therefore$ $c^2 = \dfrac{a^2+b^2}{5} = 5 \Rightarrow c = \sqrt{5}$

$\because$ $\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{16+9-5}{2\times4\times3} = \dfrac{5}{6}$

$\because$ $\sin C = \dfrac{\sqrt{11}}{6}$

$\therefore$ $\Delta = \dfrac{1}{2}$ ab sin C = $\sqrt{11}$

$\therefore$ $\Delta^2 = 11$
(B) $\because$ line joining the circumcentre and orthocentre is parallel to side AC
$\Rightarrow$ R cos B = 2R cos A cos C
$\Rightarrow$ –cos (A+C) = 2 cos A cos C
$\Rightarrow$ sin A sin C = 3 cos A cos C
$\Rightarrow$ tan A tan C = 3

(C) $\because \tan^2\dfrac{C}{2} = \dfrac{(s-a)(s-b)}{s(s-c)}$ $\because$ a = 5, b = 4 $\because$ 2s = 9 + c

$= \dfrac{(9+c-10)(9+c-8)}{(9+c)(9-c)} = \dfrac{c^2-1}{81-c^2}$

$\Rightarrow \dfrac{7}{9} = \dfrac{c^2-1}{81-c^2} \Rightarrow c^2 = 36 \Rightarrow c = 6$

(D) $\because 2a^2 + 4b^2 + c^2 = 4ab + 2ac$.
$\Rightarrow (a-2b)^2 + (a-c)^2 = 0$
$\Rightarrow$ a = 2b = c

$\because$ $\cos B = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{7}{8}$

$\therefore$ 8 cos B = 7

NUMERICAL VALUE BASED

Q.1 **(4)**
Let sides are a, b & c
a + b = p
ab = q

$c = \sqrt{p^2 - q}$
$c^2 = p^2 - q$
$c^2 = (a+b)^2 - ab$
$\therefore$ $ab = (a+b)^2 - c^2$
ab = (a + b + c) (a + b – c) = 2s(2s – 2c)

$\therefore$ $\dfrac{s(s-c)}{ab} = \dfrac{1}{4}$

$\cos^2\dfrac{C}{2} = \dfrac{1}{4} \Rightarrow \cos\dfrac{C}{2} = \dfrac{1}{2}$

$\angle C = 120°$

$\therefore$ $\Delta = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}\times q\times\dfrac{\sqrt{3}}{2} = \sqrt{3}$

$\therefore$ q = 4

Q.2 **(400)**
Let all sides are equal to x.
AB = BC = CD = AD = x.
Let AC = y BD = z

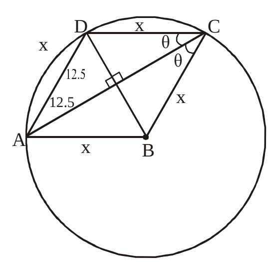
Δ of ABD $= \dfrac{abc}{4R_1} = \dfrac{xxz}{4R_1}$

Area of ABCD = 2 × Δ of ABD $= \dfrac{x^2z}{2R_1}$

Δ of ACD $= \dfrac{abc}{4R_2} = \dfrac{xxy}{4R_2}$

$\dfrac{x^2z}{2R_1} = \dfrac{x^2y}{2R_2} \Rightarrow \dfrac{z}{12.5} = \dfrac{y}{25}$

$z = \dfrac{y}{2}$, $\tan\theta = \dfrac{\frac{z}{2}}{\frac{y}{2}} = \dfrac{z}{y} = \dfrac{1}{2}$



$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} = \dfrac{4}{3}$, $\sin 2\theta = \dfrac{4}{5}$

$\dfrac{BD}{\sin 2\theta} = 2R$

BD = 2 × 12.5 × $\frac{4}{5}$ = 20 = z, y = 40

Area of Rhombus

$= 2\frac{1}{2}(AC)\cdot\frac{(BD)}{2} = \frac{(AC)(BD)}{2} = \frac{800}{2} = 400$

Q.3 **(18)**

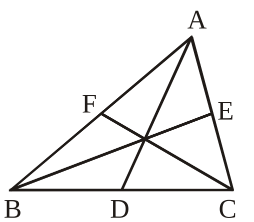
$AD = \frac{\sqrt{2b^2+2c^2-a^2}}{2}$, BE

$= \frac{\sqrt{2c^2+2a^2-b^2}}{2}$, $CF = \frac{\sqrt{2a^2+2b^2-c^2}}{2}$

Applying AM ≥ GM in AD^2, BE^2, CF^2

$$\frac{AD^2+BE^2+CF^2}{3} \geq (AD \cdot BE \cdot CF)^{2/3}$$

$$\frac{\frac{3}{4}(a^2+b^2+c^2)}{3} \geq BE^2$$



$a^2 + b^2 + c^2 \geq 4BE^2 = 4 \times 81$

$\sqrt{a^2+b^2+c^2} = 18$ Ans.

Q.4 **(11)**

$A = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\text{Area} = \frac{1}{2}\cdot bc\sin\frac{\pi}{6} = \frac{\sqrt{3}}{4}a^2$

∴ $bc = \sqrt{3}\,a^2$

$\sin B \sin C = \frac{\sqrt{3}}{4}$

$B + C = 150°$

$\sin B \sin(150° - B) = \frac{\sqrt{3}}{4}$

$\sin B\left[\frac{1}{2}\cos B + \frac{\sqrt{3}}{2}\sin B\right] \frac{\sqrt{3}}{4}$

$\frac{\sin 2B}{4} + \frac{\sqrt{3}}{4}(1-\cos 2B) = \frac{\sqrt{3}}{4}$

$\Rightarrow \sin 2B - \sqrt{3}\cos 2B = 0 \Rightarrow \tan 2B = \sqrt{3}$

$2B = 68°$ or $240°$

∴ B = 30° or 120°

When B = 30°, C = 120°

When B = 120°, C = 30°

But b > c ⇒ B > C

∴ A = 30°, B = 120°, C = 30°

$\text{cosec}^2 A + \sec^2 B + \cot^2 C = 4 + 4 + 3 = 11.$ **Ans.**]

Q.5 **(7)**

$\cos A = \frac{9+16-4}{2\cdot 3\cdot 4} = \frac{7}{8}$

$s = \frac{9}{2}$

$D = \sqrt{s(s-a)(s-b)(s-c)} = \frac{3\sqrt{15}}{4}$

$HA = 2R\cos A = 2\cdot\left(\frac{abc}{4\Delta}\right)\cdot\cos A = 2\left(\frac{2\cdot 3\cdot 4}{4\cdot\frac{3\sqrt{5}}{4}}\right)$

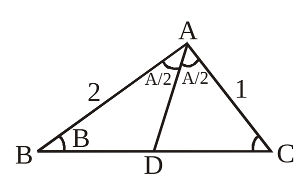
$\frac{7}{8} = \frac{14}{\sqrt{15}}$

$\Rightarrow \frac{\sqrt{15}\,HA}{2} = 7.$ Ans.

Q.6 **(9)**

AD = BD

∴ $\frac{A}{2} = B$



A = 2B

$C = \pi - (A + B) = \pi - 3B$

Using sine Rule, in triangle ABC

$$\frac{\sin B}{1} = \frac{\sin(\pi - 3B)}{2}$$

$\Rightarrow 2\sin B = \sin 3B = (3\sin B - 4\sin^3 B)$

$\Rightarrow \quad 4\sin^2 B = 1 \Rightarrow \sin B = \frac{1}{2}$

$\therefore \quad B = 30°, A = 60°, C = 90°.$

$\Delta = \frac{1}{2} \times 2 \times 1 \times \sin A = \sin A = \frac{\sqrt{3}}{2}$

$12\Delta^2 = 12 \times \frac{3}{4} = 9.$ **Ans.**]\

Q.7 (**0**)

$B = 2A, C = 4A$

$A + B + C = \pi$

$\therefore \quad A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$

$$\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = \frac{1}{2R}\left[\frac{1}{\sin B} + \frac{1}{\sin C} - \frac{1}{\sin A}\right]$$

$$= \frac{1}{2R\sin A\sin B\sin C}$$

$[\sin A\sin C + \sin A\sin B - \sin B\sin C]$

$$= \frac{1}{2R\sin A\sin B\sin C}$$

$$\left[\sin\left(\frac{\pi}{7}\right)\sin\left(\frac{4\pi}{7}\right) + \sin\left(\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{7}\right) - \sin\left(\frac{2\pi}{7}\right)\sin\left(\frac{4\pi}{7}\right)\right]$$

$$= \frac{1}{4R\sin A\sin B\sin C}\left[\left(\cos\frac{3\pi}{7} - \cos\frac{5\pi}{7}\right) + \left(\cos\frac{\pi}{7} - \cos\frac{3\pi}{7}\right) - \left(\cos\frac{2\pi}{7} - \cos\frac{6\pi}{7}\right)\right]$$

$= 0.$ **Ans.**

Q.8 **(42)**

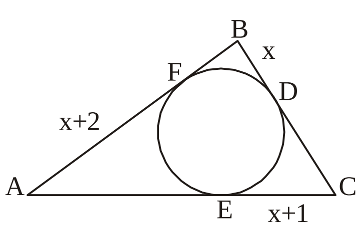
Suppose that BD = x,

CE = x + 1 and AF = x + 2. Then

CD = CE = x + 1

AE = AF = x + 2

BF = BD = x



Hence $a = BC = x + x + 1 = 2x + 1$

$b = CA = x + 1 + x + 2 = 2x + 3$

$c = AB = x + 2 + x = 2x + 2$

$s = \frac{a+b+c}{2} = 3x + 3$

$s - a = x + 2, s - b = x, s - c = x + 1$

Since $r = \frac{\Delta}{s} = \frac{1}{s}\sqrt{s(s-a)(s-b)(s-c)}$

we get $4 = \sqrt{\frac{(x+2)x(x+1)}{3x+3}}$ i.e.

$16 = \frac{x(x+2)}{3} \Rightarrow x^2 + 2x = 48$

i.e. $(x + 8)(x - 6) = 0$; $x = 6$ or -8 but $x \neq -8 \Rightarrow x = 6$

Hence perimeter $= 6(x + 1) = 42$]

Q.9 **(42)**

Given, the equation $ax^2 + bx + c = 0$ has equal roots, so $b^2 = 4ac$ .... (1)

As, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{a^2 + c^2 - 4ac}{2ac}$ (Using equation (1))

$\Rightarrow \quad 2\cos^2\frac{B}{2} - 1 = \frac{a^2 + c^2}{2ac} - 2 \Rightarrow 2\cos^2\frac{B}{2}$

$= \frac{a^2 + c^2}{2ac} - 1$

$\Rightarrow \quad \cos^2\frac{B}{2} = \frac{a^2 + c^2}{4ac} - \frac{1}{2}$

$= \frac{1}{4}\left(\frac{a}{c} + \frac{c}{a} - 2\right)$

As, $0 < \cos^2\frac{B}{2} < 1 \Rightarrow 0$

$< \frac{1}{4}\left(\frac{a}{c} + \frac{c}{a} - 2\right) < 1 \Rightarrow 0 < \left(\frac{a}{c} + \frac{c}{a} - 2\right) < 4$

$\Rightarrow \quad 2 < \frac{a}{c} + \frac{c}{a} < 6$ .... (2)

Also, $\frac{\sin A}{\sin C} + \frac{\sin C}{\sin A} = \frac{a}{c} + \frac{c}{a}$

(Using sine law in ΔABC) ....(3)

$\therefore$ From (2) and (3), we get

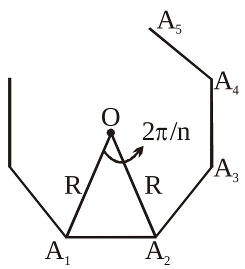
$$2 < \frac{\sin A}{\sin C} + \frac{\sin C}{\sin A} < 6.$$

Hence, possible integers in the range = 3, 4, 5.
So, sum of integers = 3 + 4 + 5 = 12. **Ans.**]

Q.10 **(7)**

ΔOA_1A_2

$$\cos\left(\frac{2\pi}{n}\right) = \frac{R^2 + R^2 (A_1A_2)^2}{2 \cdot R \cdot R}$$



$$(A_1A_2)^2 = 2R^2\left(1 - \cos\frac{2\pi}{n}\right)$$

$$A_1A_2 = 2R\sin\left(\frac{\pi}{n}\right)$$

Similarly, $A_1A_3 = 2R\sin\frac{2\pi}{n}$, $A_1A_n = 2R\sin\frac{3\pi}{n}$

$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$$

$$\frac{1}{2R\sin\frac{\pi}{n}} = \frac{1}{2R\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{2R\sin\left(\frac{3\pi}{n}\right)}$$

$$\therefore \quad \sin\frac{2\pi}{n}, \sin\frac{3\pi}{n} = \sin\frac{\pi}{n}$$

$$\left(\sin\frac{3\pi}{n} + \sin\frac{2\pi}{n}\right)$$

$$\Rightarrow 2\cos\frac{\pi}{n}\sin\frac{3\pi}{n} = \sin\frac{3\pi}{n} + \sin\frac{2\pi}{n}$$

$\Rightarrow$

$$\sin\left(\frac{4\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right)$$

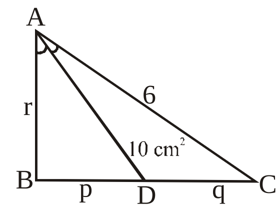
$$\Rightarrow \sin\left(\frac{4\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)$$

$\therefore$ n = 7. **Ans.**

KVPY
PREVIOUS YEAR'S

Q.5 (D)

From angle bisector theorem



$$\frac{r}{6} = \frac{p}{q}$$

q r = 6 p ....(1)

Area of $\Delta ADC = 10\ cm^2$

$$\frac{1}{2}(DC)(AB) = 10$$

$$\frac{1}{2}(q)(r) = 10$$

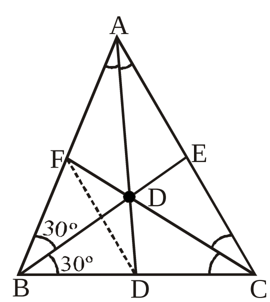
q r = 20

From (1)

$\Rightarrow$ 20 = 6 p

$$p = \frac{20}{6} = \frac{10}{3}$$

Q.6 (B)



$$\angle AIC = 180^\circ - \left(\frac{A+C}{2}\right)$$

$$= 180^\circ - \left(\frac{A+C}{2}\right)$$

$$= 90 + \frac{B}{2}$$

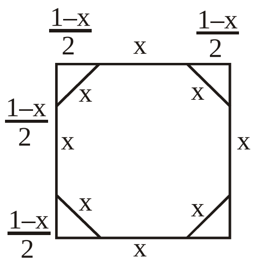
$$\text{Here}: -90 + \frac{B}{2} + B = 180^\circ$$

$B = 60^o$

This will be case of equilateral Δ

$\Rightarrow \angle IFD = 30^o$

Q.7 (B)

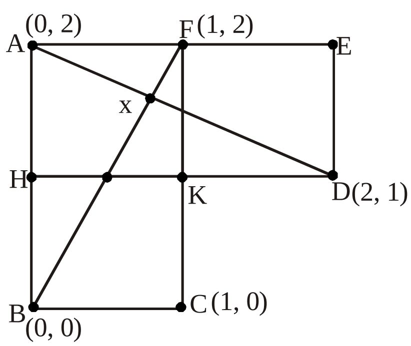


$$\text{Here } 2\left(\frac{1-x}{2}\right)^2 = x^2$$

$$(1-x)^2 = 2x^2$$

$$x = \sqrt{2} - 1$$

Q.8 (B)



Equation of BF

y = 2x

equation of AD

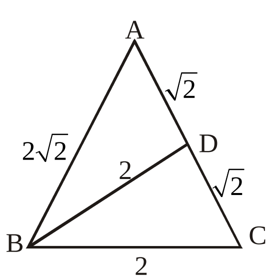
x + 2y = 4

$$x\left(\frac{4}{5}, \frac{8}{5}\right)$$

$$\frac{ar(\Delta AFX)}{ar(\Delta ABF)} = \frac{\frac{1}{2}\times 1\times\left(2-\frac{8}{5}\right)}{\frac{1}{2}\times 1\times 2} = \frac{1}{5}$$

Q.9 (2)

Q.10 (C)



We know

$$AB^2 + BC^2 = 2(CD^2 + BD^2)$$

$$AB^2 + 4 = 2\left(\frac{AB^2}{4} + 4\right)$$

$$AB^2 + 4 = \frac{AB^2}{2} + 8$$

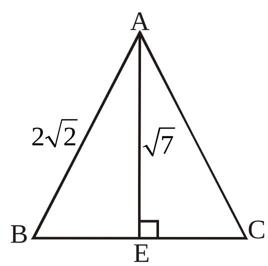
$$\frac{AB^2}{2} = 4$$

$$AB^2 = 8$$

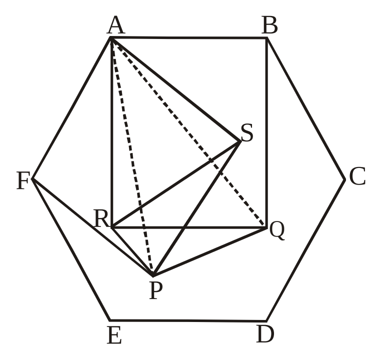
$$AB = 2\sqrt{2}$$

Now

$$\text{Area} = \frac{1}{2}\times 2\times\sqrt{7} = \sqrt{7} \text{ square unit.}$$



Q.11 (D)



In ΔAPQ

$AP = AQ = \sqrt{2}$, $\angle APQ = 30°$

In ΔSRP

$SR = SP = 1, \angle RSP = 30°$

$\angle FAB = 120°$

$\angle BAS = \angle FAB - \angle FAS = 120° - 90° = 30°$

$\angle SAR = \angle BAR - \angle BAS = 60^o$

In ΔARS

$$\cos 60° = \frac{1+1-SR^2}{2.1.1} \quad [\because AR = AS = 1]$$

$\Rightarrow SR = 1$

Now, $\angle RSP = \angle ASP - \angle ASR$

$= 90° - 60° = 30°$ [$\because$ ASR is equilateral]

$$\text{Now, from } \Delta SRP \Rightarrow RP = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

In ΔAPR

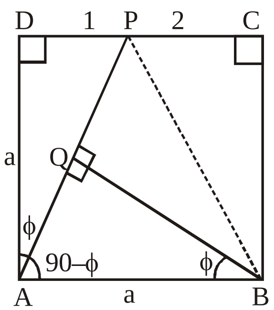
$$\cos\angle RAP = \frac{\left(\sqrt{2}\right)^2 + 1^2 - PR^2}{2\sqrt{2}}$$

$\Rightarrow \cos\angle RAP = 15°$

$\angle PAQ = \angle RAQ - \angle RAP = 45° - 15° = 30°$

$$\text{Now, } \frac{ar(\Delta APQ)}{ar(\Delta SRP)} = \frac{\frac{1}{2}\times\sqrt{2}\times\sqrt{2}\sin 30°}{\frac{1}{2}\times 1\times 1\times \sin 30°} = 2$$

Q.12 (D)



$CP = \frac{2}{3}a$ $\quad\quad$ $PD = \frac{a}{3}$

Let $\angle PAD = \phi$ $\quad\quad$ $\tan\phi = \frac{1}{3}(\text{In } \Delta APD)$

Now, $\angle DAP = \angle QBA = \phi$

Required ratio

$$= \frac{\text{area of PQBC}}{a^2} = \frac{a^2 - (\text{area } \Delta ADP + \text{area of } \Delta AQB)}{a^2}$$

$$\Rightarrow 1 - \left(\frac{1}{6} + \frac{3}{20}\right) = \frac{41}{60}$$

Q.13 (C)

Case (I)

$b + 5 = 3b - 2$

$\Rightarrow b = \frac{7}{2}$

So sides are $\frac{17}{2}, \frac{17}{2}, \frac{5}{2}$

Case (II)

$b + 5 = 6 - b \Rightarrow b = \frac{1}{2}$

Sides $\frac{11}{2}, \frac{-1}{2}, \frac{11}{2}$ Not possible

Case (III) $3b - 2 = 6 - b$

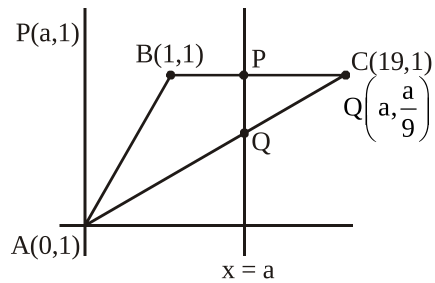
$4b = 8$

$b = 2$

7, 4, 4

two cases are possible

Q.14 (A)

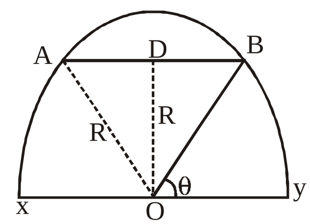


area of PQC = $\frac{1}{2}$ area of ABC

$$\frac{1}{2}(9-a)\left(1-\frac{a}{9}\right) = \frac{1}{2}\times\frac{1}{2}(8\times 1)$$

$(9-a)^2 = 36 \Rightarrow a = 3$

Q.15 (A)



$OD = R\sin\theta$

$AB = 2R\cos\theta$

$$r_{OAB} = \frac{ar(\Delta OAB)}{\text{semi}-\text{perimeter}}$$

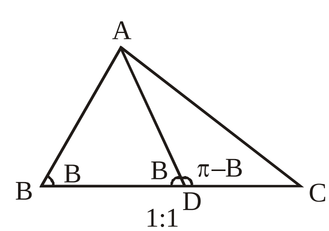
$$= \frac{\frac{1}{2}\times OD\times AB}{\frac{2R+AB}{2}} = \frac{\frac{1}{2}\times R\sin\theta 2R\cos\theta}{\frac{2R+2R\cos\theta}{2}}$$

$$r_{OAB} = \frac{R\sin\theta\cos\theta}{(1+\cos\theta)}$$

$$\frac{dr_{OAB}}{d\theta} = \frac{(1+\cos\theta)\cos 2\theta - \sin\theta\cos\theta(-\sin\theta)}{(1+\cos\theta)^2} = 0$$

at $\cos\theta = \frac{\sqrt{5}-1}{2}$

Q.16 (D)



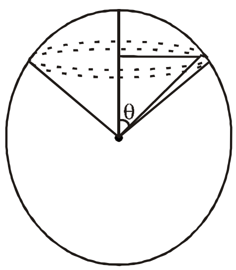
Using M – N Rule

(1 + 1) cot (π – B) = 1.cot B – cot C

3 cot B = cot C

$\frac{\tan B}{\tan C} = 3$

Q.17 (D)



AP = R sinθ

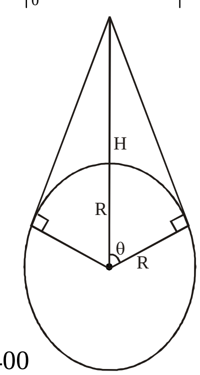
area of ring = (2π Rsinθ). Rdθ

Total area required, $\pi R^2 = \left|\int_0^{\theta} 2\pi R^2 \sin\theta d\theta\right|$

$\frac{1}{2} = 1 - \cos\theta$

Now,

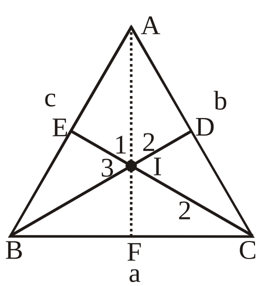
$\cos\theta = \frac{1}{2}$



$\cos\theta = \frac{R}{R+H}$

$\frac{1}{2} = \frac{R}{R+H}$ = R = H = 6400

Q.18 (B)



I deivides AF

b + c : a

I divided BD

c + a : b

I divides CE in

a + b : c

$\frac{c+a}{b} = \frac{3}{2}; \frac{a+b}{c} = \frac{2}{1}$

2c + 2a = 3b. $\Rightarrow a + b = 2c$

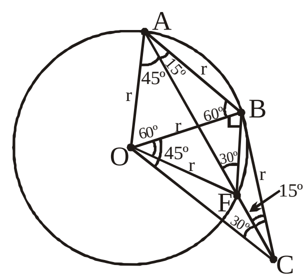
$\Rightarrow a + b + 2a = 3b$ $a + \frac{3a}{2} = 2c$

$\Rightarrow 3a = 2b$ $\frac{5}{2}a = 2c$

$\Rightarrow b = \frac{3b}{2}$ $c = \frac{5}{4}a$

Now $\frac{b+c}{a} = \frac{\frac{3a}{2} + \frac{5a}{4}}{a} = \frac{6+5}{4} = \frac{11}{4}$

Q.19 (B)



∴ ΔOAB should be equilateral

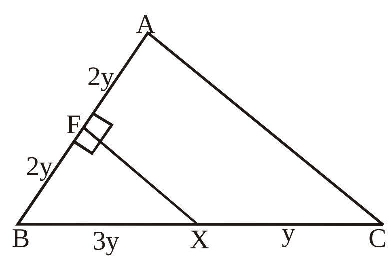
Now ∠OBC = 90º & AB = BC

$\Rightarrow \angle BAC = \angle BCA = 15^\circ$

∴ ∠BOF = 30° & ∠BOC = 45° {ΔOBC is right isosceles}

$\therefore \frac{\angle BOF}{\angle BOC} = \frac{30^\circ}{45^\circ} = \frac{2}{3}$ {∠BOF = 2∠BAF}

Q.20 (C)



$\because \cos B = \frac{2}{3}$

Also $\cos B = \frac{16y^2 + 16y^2 - AC^2}{2.4y.4y}$

$\frac{2}{3} = \frac{32y^2 - AC^2}{32y^2}$

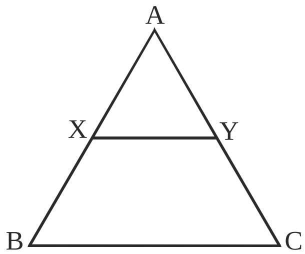
$64y^2 = 96y^2 - 3AC^2$

$3AC^2 = 32y^2$

$AC = \frac{4\sqrt{2}}{\sqrt{3}}\ y$

$$\therefore \frac{BC}{AC} = \frac{4y}{\left(\frac{4\sqrt{2}y}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{\sqrt{2}}$$

Q.21 (D)



Clearly ar (BCX) = ar (BCY)

{Δs between parallel lines & same base}

$\Rightarrow$ [BCX] = [BCY]

(I) is true.

Check

(II) ar (ΔACX) = $\frac{1}{2}$ AC.AX sin A

ar(ΔABY) = $\frac{1}{2}$ AB.AY sin A.

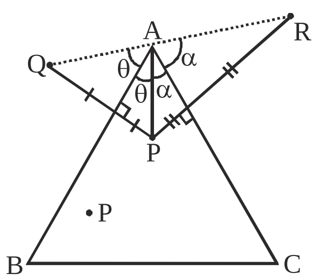
ar (ΔAXY) = $\frac{1}{2}$ AX.AY sin A

ar (ΔABC) = $\frac{1}{2}$ AB.AC sin A.

Clearly [ACX].[ABY] = [AXY].[ABC]

(II) is true.

Q.22 (C)

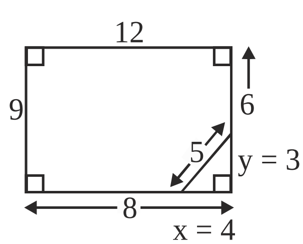


$2\theta + 2\alpha = 180^o$

$\Rightarrow \theta + \alpha = 90^o$

$\Rightarrow \angle A = 90^o$

Q.23 (4)



Clearly x = 4 = 12 – 8

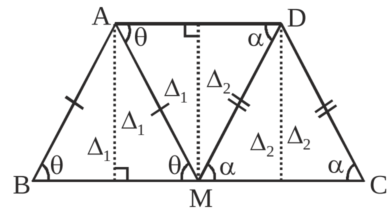
& y = 3

area of rectangle = 12 × 9 – area of Δ

$$= 108 - \frac{1}{2} \times 3 \times 4 = 102$$

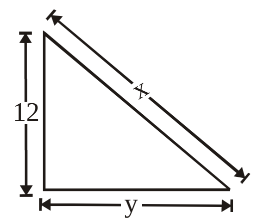
$$\therefore \frac{ar(pentagon)}{ar(rectangle)} = \frac{102}{108} = \frac{17}{18}$$

Q.24 (B)



$$\frac{ar(ABCD)}{ar(AMD)} = \frac{3\Delta_1 + 3\Delta_2}{\Delta_1 + \Delta_2} = \frac{3}{1}$$

Q.25 (D)



Clearly

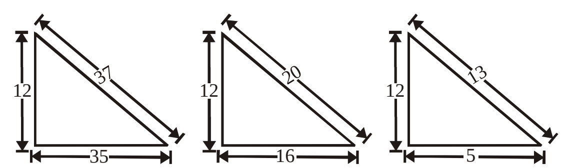
$x^2 - y^2 = 144$

$(x - y)(x + y) = 144$

$x, y \in I$

So factorsize 144 into two even factors

$x + y = 72$	$x + y = 36$	$x + y = 18$
$x - y = 2$	$x - y = 9$	$x - y = 18$
$x = 37$	$x = 20$	$x = 13$
$y = 35$	$y = 16$	$y = 5$



r = Δ/s r = Δ/s r = Δ/s

$$r = \frac{1/2 \times 12 \times 35}{\left(\frac{12 + 35 + 37}{2}\right)} \qquad r = \frac{1/2 \times 12 \times 16}{\left(\frac{16 + 20 + 12}{2}\right)}$$

r = 2

$$r = \frac{12 \times 35}{84} \qquad r = \frac{12 \times 16}{48}$$

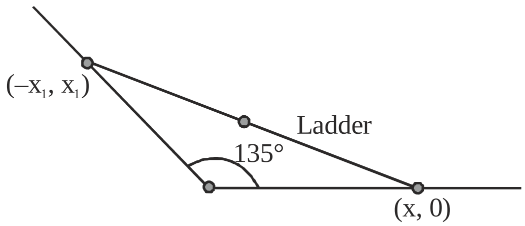
r = 5 r = 4

Q.26 (C)

Formed triangle will be right angle whose sides are 3, 4, 5

So. circumradius = $\frac{\text{length of hypotenuse}}{2} = 2.5$

Q.27 (A)



Mid point (h, k) $= \left(\frac{x - x_1}{2}, \frac{x_1}{2}\right)$

Now $(x + x_1)^2 + x_1^2 = \ell^2$

As $2h + 4k = x + x_1$, $2k = x$,

So required locus is

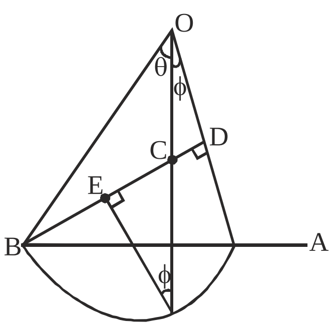
$4(h + 2k)^2 + 4k^2 = \ell^2$

$$h^2 + 5k^2 + 4hk = \frac{\ell^2}{4}$$

$$x^2 + 5y^2 + 4xy = \frac{\ell^2}{4}$$

Whose area is $\frac{\pi\ell^2}{4}$

Q.28 (D)



$CF = r(1 - \cos\theta \sec\phi)$

$EC = r(\sin\phi - \cos\theta \tan\theta)$

$CD = r\cos\theta \tan\phi$

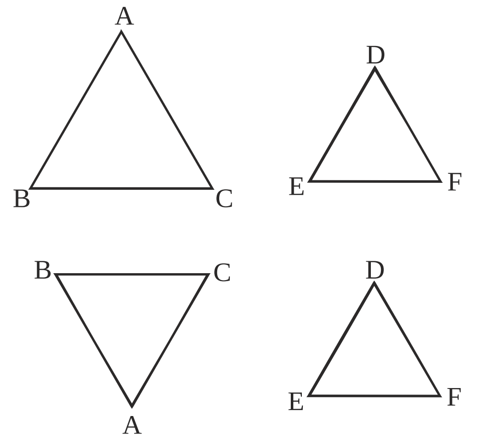
$EC + CD = ED$

$$r = \sin\phi = \frac{r \sin\theta}{2}$$

$$\phi = \sin^{-1}\left(\frac{\sin\theta}{2}\right)$$

Q.29 (C)

Let triangle T is DEF possibilities

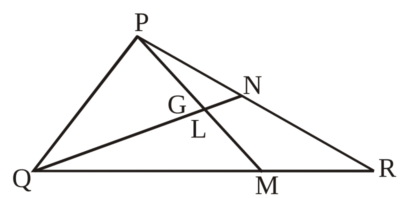


$\therefore$ A can take two positions if $\Delta ABC \sim \Delta DEF$

We can arrange order of A, B, C in 3! = 6 ways

Total positions which A can take = 6 × 2 = 12

Q.30 (C)



Let QR = p, PR = q, PQ = r

Given $p^2 + q^2 = 5r^2$

$$\text{Now} \quad QG^2 + GM^2 = \left(\frac{2\,QN}{3}\right)^2 + \left(\frac{PM}{2}\right)^2$$

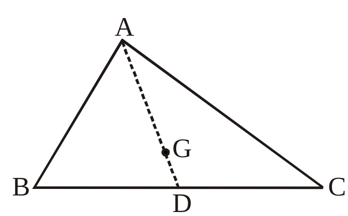
$$= 4\frac{QN^2}{9} + \frac{PM^2}{9}$$

$$= \frac{1}{9}\left[4.\frac{1}{4}\left(2r^2 + 2p^2 - q^2\right) + \frac{1}{4}\left(2r^2 + 2q^2 - p^2\right)\right]$$

$$= \frac{p^2}{4} = QM^2$$

Hence Angle QGM is 90º.

Q.31 (C)



As AD is medium $\Rightarrow AD < \frac{AB + AC}{2}$

$$\Rightarrow \ell < \frac{b + c}{2}$$

Similarly $m < \frac{c + a}{2}$ and $n < \frac{a + b}{2}$

$\Rightarrow \ell + m + n < a + b + c$

$\Rightarrow \frac{\ell+m+n}{a+b+c}<1$ (i)

Also in the Δ BGC

$BG+GC>BC$

$\Rightarrow \frac{2}{3}(m+n)>a$

Similarly $\frac{2}{3}(n+\ell)>b$

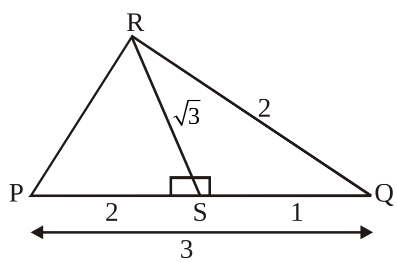
and $\frac{2}{3}(\ell+m)>c$

Hence $\frac{4}{3}(\ell+m+n)>a+b+c$

$\frac{\ell+m+n}{a+b+c}>\frac{3}{4}$ (ii)

By (i) and (ii) $\frac{\ell+m+n}{a+b+c}\in\left(\frac{3}{4},1\right)$

Q.32 (C)



$PS = QR$

$PS + SQ = 3$

in ΔRSQ $SQ = 3 - PS = 3 - QR$

$QR^2 = RS^2 + SQ^2$

$QR^2 = 3 + (3 - QR)^2$

$QR^2 = 3 + 9 + QR^2 - 6QR$

$6QR = 12$

$QR = 2$

$SQ = 1$ $PS = 2$

in ΔRSP

$PR^2 = RS^2 + PS^2$

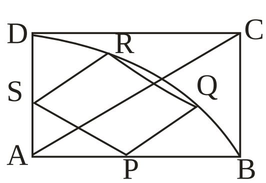
$= 3 + 4$

$PR^2 = 7$

$PR = \sqrt{7}$

Q.33 (D)

Let A (0, 0), B(1, 0), C(1, 1) & D(0, 1)



$\Rightarrow$ Area ABCD = 1

Again let Q $(\cos\alpha, \sin\alpha)$ & R $(\cos\beta, \sin\beta)$

$\Rightarrow$ coordinate of $P(\cos\alpha - \sin\alpha, 0)$ & $S(0, \sin\beta - \cos\beta)$

PQRS is a square $\Rightarrow PQ \perp OR$

$\Rightarrow$ slope of QR = –1 = slope of SP

$\Rightarrow \frac{\sin\beta-\sin\alpha}{\cos\beta-\cos\alpha}=-1=\frac{\sin\beta-\cos\beta}{\sin\alpha-\cos\alpha}$

$\Rightarrow \sin\beta - \sin\alpha = -\cos\beta + \cos\alpha$

$\Rightarrow \sin\beta - \cos\beta = \sin\alpha + \cos\alpha$ .....(i)

and $\sin\alpha + \sin\beta = \cos\alpha + \cos\beta$ .....(ii)

$\Rightarrow \cos\alpha = \sin\beta$

$\Rightarrow \cos\alpha = \cos(90 - \beta)$

$\Rightarrow \alpha + \beta = 90^o$

Also PQ = QR

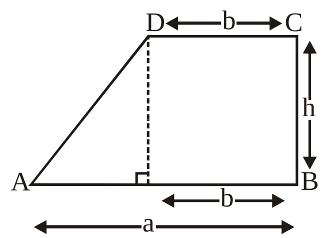
$\Rightarrow \tan\alpha = \frac{1}{2}$

Area of PQRS $= 2\sin^2\alpha = 2\left(\frac{1}{5}\right)$

$\frac{\text{Area of PQRS}}{\text{Area of ABCD}}=\frac{2/5}{1}=\frac{2}{5}$

Q.34 (B)

Given : $\frac{1}{2}(a+b)\times h=12$



$(a+b)\times h = 24$

24×1

12×2

6×4

8×3

In height angle. ΔAED possible height for integer sides.

h = 4, 3

Case I :- When h = 4

Then possible triplet (3, 4, 5)

i.e. DE = 4, AE = 3, AD = 5

if AE = 3, 2b = 3

b = 3/2

(Not possible because $b \in 1$)

Case II :- When h = 3

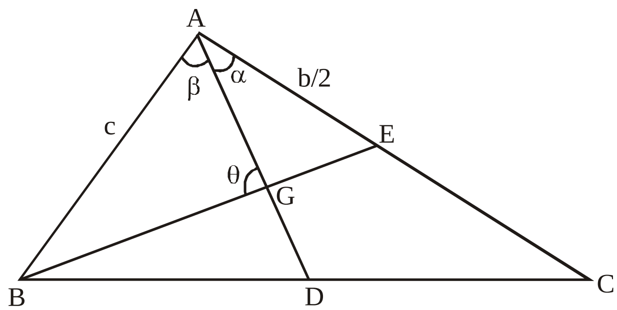
Then AE = 4, 2b = 4

b = 2

$\therefore$ CD = 2, AB = 6

$\therefore$ |AB – CD| = 4

Q.35 (A)



Given $\frac{\cot\alpha}{\cot\beta} = \frac{2}{1}$

$\Rightarrow \cot\alpha = 2\cot\beta$

$$\Rightarrow \frac{AG^2 + AE^2 - GE^2}{2(AG)(AE)\sin\alpha} = 2\left[\frac{AB^2 + AG^2 - BG^2}{2(AB)(AG)\sin\beta}\right]$$

$$\Rightarrow \frac{\frac{2b^2 + 2c^2 - a^2}{9} + \frac{b^2}{4} - \frac{2a^2 + 2c^2 - b^2}{36}}{\frac{4}{3}ar(\Delta ADC)}$$

$$= \frac{c^2 + \frac{2b^2 + 2c^2 - a^2}{9} - \frac{2a^2 + 2c^2 - b^2}{9}}{\frac{4}{3}ar(\Delta ADC)}$$

$\Rightarrow 4(2b^2 + 2c^2 - a^2) + 9b^2 - 2a^2 - 2c^2 + b^2 = 36c^2 + 4(3b^2 - 3a^2)$

$\Rightarrow a^2 + b^2 = 5c^2$

$$\cos\theta = \frac{AG^2 + BG^2 - c^2}{2(AG)(BG)}$$

$$AG^2 + BG^2 - c^2 = \frac{2b^2 + 2c^2 - a^2}{9} + \frac{2a^2 + 2c^2 - b^2}{9} = c^2$$

$$= \frac{a^2 + b^2 - 5c^2}{9} = 0$$

$\Rightarrow \theta = 90°$

Q.36 **(C)**

In an equilateral triangle R = 2r

$\Rightarrow \quad \frac{R}{r} = 2, \forall a$

Q.37 **(B)**

Substitute $C^2 = 2ab$ in $a^2 + c^2 = 3ab^2$

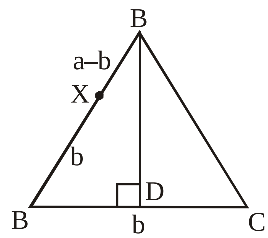
$\Rightarrow a^2 + 2ab - 3ab^2 = 0$

$\Rightarrow (a - b)(a + 3b) = 0$

$\Rightarrow a = b \;\&\; c = a\sqrt{2}$

$\Rightarrow \angle A = \angle B = 45°$

Q.38 **(B)**



Let AC = b & AB = BC = a

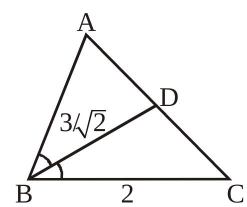
Given $\frac{b}{a-b} = \frac{a}{b} \Rightarrow b^2 = a^2 - ab \Rightarrow \frac{b}{a} = \frac{-1+\sqrt{5}}{2}$

Let D be foot at perpendicular from B

$$\sin(\angle ABD) = \frac{b}{2a} = \frac{\sqrt{5}-1}{4} \Rightarrow \angle ABD = 18°$$

$\therefore \quad \angle ABC = 36°$

Q.39 **(B)**



$$\frac{2ac}{a+c}\cos\frac{B}{2} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \frac{4c}{2+c}\left[\frac{4 + \frac{9}{2} - 1}{2\times 2\times \frac{3}{\sqrt{2}}}\right] = \frac{3}{\sqrt{2}}$$

$\Rightarrow c = 3$

Now, $\frac{c}{a} = \frac{AD}{DC} \Rightarrow AD = \frac{3}{2}$

$\Rightarrow b = \frac{5}{2}$

$\Rightarrow$ perimeter $= \frac{15}{2}$

Q.40 **(B)**

$$m^2 = \frac{2b^2 + 2c^2 - a^2}{4}$$

$\Rightarrow 144 + 64 = 2[b^2 + (b-2)^2]$

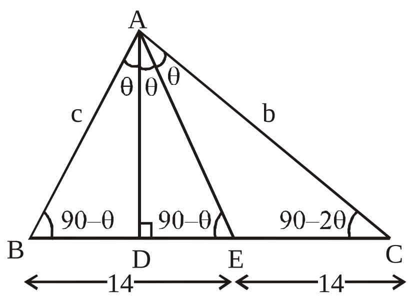
$\Rightarrow 104 = 2b^2 - 4b + 4$

$\Rightarrow b^2 - 2b - 50 = 0$

$\Rightarrow (b-1)^2 = 51$

$\Rightarrow b = 1 + \sqrt{51} \in (8, 9)$

Q.41 (**A**)



ΔABE is isosceles $\Rightarrow BD = DE = 7$

$\Delta ADC : \tan(90 - 2\theta) = \dfrac{AD}{21}$ .... (1)

$\Delta ADE : \tan(90 - \theta) = \dfrac{AB}{7}$ .... (2)

Divide $\dfrac{\tan\theta}{\tan 2\theta} = \dfrac{1}{3} \Rightarrow \dfrac{1\tan^2\theta}{2} = \dfrac{1}{3}$

$1 - \tan 2\theta = \dfrac{2}{3} \Rightarrow \tan = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30^o$

$\Delta ABD : \cos(90 - \theta) = \dfrac{BD}{C} = \sin\theta$

$C = 7\ \text{cosec}\theta = 14$

$\Delta ADC : \cos(90 - 2\theta) = \dfrac{DC}{b} = \sin 2\theta$

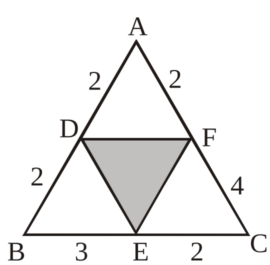
$b = 21\text{cosec}2\theta = 21\text{cosec}\dfrac{\pi}{3}$

$b = \dfrac{42}{\sqrt{3}} = 14\sqrt{3}$

$b + c = 14\sqrt{3} + 14$

$[b + c] = 38$

Q.42 **(C)**



$\dfrac{\text{area}(\Delta DEF)}{\text{area}(\Delta ABC)} =$

Let area $(\Delta ABC) = \Delta$

$$\frac{\text{area}(\Delta BED)}{\text{area}(\Delta ABC)} = \frac{\frac{1}{2}.6\sin B}{\frac{1}{2}.20\sin B} = \frac{3}{10}$$

$$\frac{\text{area}(\Delta ADF)}{\text{area}(\Delta ABC)} = \frac{\frac{1}{2}.4\sin A}{\frac{1}{2}.24\sin A} = \frac{1}{6}$$

$$\frac{\text{area}(\Delta CEF)}{\text{area}(\Delta ABC)} = \frac{\frac{1}{2}.8\sin C}{\frac{1}{2}.30\sin C} = \frac{4}{15}$$

$\text{area}(\Delta BED) = \dfrac{3}{10}\Delta$

$\text{area}(\Delta ADF) = \dfrac{\Delta}{6}$

$\text{area}(\Delta CEF) = \dfrac{4\Delta}{15}$

$\text{area}(\Delta DEF) = \Delta - \left(\dfrac{3}{10} + \dfrac{1}{6} + \dfrac{4}{15}\right)\Delta$

$= \Delta - \left(\dfrac{27 + 15 + 24}{90}\right)\Delta$

$= \Delta - \left(\dfrac{66}{90}\right)\Delta = \dfrac{24\Delta}{90}$

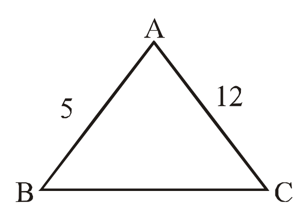
$\dfrac{\text{area}(\Delta DEF)}{\text{area}(\Delta ABC)} = \dfrac{24}{90} = \dfrac{4}{15}$

JEE MAINS

PREVIOUS YEAR'S

Q.1 **(15)**

$\Delta = \dfrac{1}{2}.5.12.\sin A = 30$



$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$r = \dfrac{\Delta}{S} = \dfrac{30}{15} = 2$

$2R + r = 15$

Q.2 (1)

$4(x-1) = \log_2(x-3)$

$2^{4(x-1)} = (x-3)$ here $x \ge 3$

So no solution

Q.3 **(2)**

Q.4 (1)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(C)**

$a = 2 = QR$

$b = \dfrac{7}{2} = PR$

$c = \dfrac{5}{2} = PQ$

$s = \dfrac{a+b+c}{2} = \dfrac{8}{4} = 4$

$\dfrac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \dfrac{2\sin P(1-\cos P)}{2\sin P(1+\cos P)} =$

$\dfrac{1-\cos P}{1+\cos P} = \dfrac{2\sin^2\frac{P}{2}}{2\cos^2\frac{P}{2}} = \tan^2\dfrac{P}{2}$

$= \dfrac{(s-b)(s-c)}{s(s-a)} = \dfrac{(s-b)^2(s-c)^2}{\Delta^2} =$

$\dfrac{\left(4-\frac{7}{2}\right)^2\left(4-\frac{5}{2}\right)^2}{\Delta^2} = \left(\dfrac{3}{4\Delta}\right)^2$

Q.2 (B,D)

Q.3 **(B)**

$a + b = x$

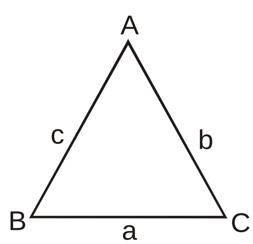
$ab = y$

$x^2 - c^2 = y$

$(a+b)^2 - c^2 = ab$

$a^2 + b^2 + ab = c^2$

$a^2 + b^2 - c^2 = -ab$



$\dfrac{a^2+b^2-c^2}{2ab} = \dfrac{7}{2}$

$\cos C = \dfrac{-1}{2}$

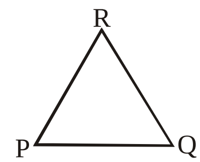
$C = \dfrac{2\pi}{3}$

$\dfrac{r}{R} = \dfrac{\Delta \times 4\Delta}{s \times abc} = \dfrac{4 \times \frac{1}{4}a^2b^2\sin^2 C}{(a+b+c)abc} = \dfrac{3ab}{4c(x+c)}$

$= \dfrac{3y}{4c(x+c)}$

Comprehension # 1 (Q. No. 4 to 5)

Q.4 **(A)**

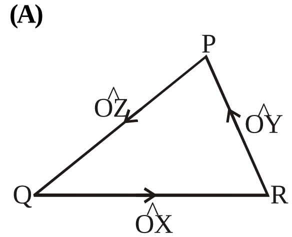


$\cos(P+Q) + \cos(Q+R) + \cos(R+P) = -\cos R - \cos P - \cos Q$

In any Δ, max of $\cos P + \cos Q + \cos R = \dfrac{3}{2}$

So minimum value of the given expression is $-\dfrac{3}{2}$

Q.5 **(A)**



$\cos R = -\hat{OX}.\hat{OY}$

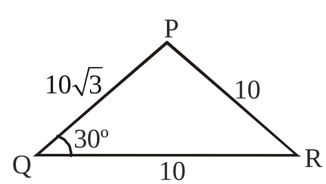
$\Rightarrow |\cos R| = |\hat{OX}.\hat{OY}|$

$\left|\hat{OX} \times \hat{OY}\right| = |\sin R| = |\sin(\pi-(P+Q))| = \sin(P+Q)|$

$= \sin(P+Q)$

Q.6 **(B,C,D)**

$\cos 30^o = \dfrac{(10\sqrt{3})^2 + (10)^2 - (PR)^2}{2 \times 10\sqrt{3} \times 10}$



$\Rightarrow PR = 10$

$\because QR = PR \Rightarrow \angle PQR = \angle QPR$

$\angle QPR = 30^\circ$

(B) area of Δ PQR = $\frac{1}{2} \times 10\sqrt{3} \times 10 \times \sin 30^\circ$

$= \frac{1}{2} \times 10 \times 10\sqrt{3} \times \frac{1}{2} = 25\sqrt{3}$

$\angle QRP = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$

(C) $r = \frac{\Delta}{S} = \frac{25\sqrt{3}}{\left(\frac{10+10+10\sqrt{3}}{2}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}}$

$= 5\sqrt{3}.(2-\sqrt{3}) = 10\sqrt{3} - 15$

(D) $R = \frac{a}{2\sin A} = \frac{10}{2\sin 30^\circ} = 10$

$\therefore$ Area $= \pi R^2 = 100\pi$

Q.7 (B,C,D)

$\frac{\sin P}{\sqrt{3}} = \frac{\sin Q}{1} = \frac{1}{2R} = \frac{1}{2}$

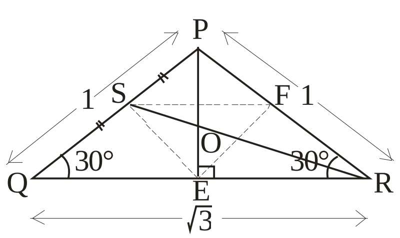
$\Rightarrow P = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ and $Q = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Since $p > q \Rightarrow P > Q$

So, if $P = \frac{\pi}{3}$ and $Q = \frac{\pi}{6} \Rightarrow R = \frac{\pi}{2}$ (not possible)

Hence, $P = \frac{2\pi}{3}$ and $Q = R = \frac{\pi}{6}$

$r = \frac{\Delta}{s} = \frac{\frac{1}{2}(1)(\sqrt{3})\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}+2}{2}\right)} = \frac{\sqrt{3}}{2}(2-\sqrt{3})$



Now, area of ΔSEF = $\frac{1}{4}$ area of ΔPQR

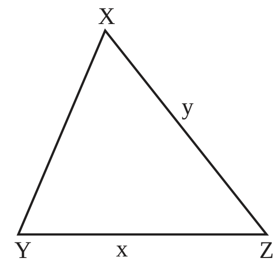
$\Rightarrow$ area of ΔSOE = $\frac{1}{3}$ area of ΔSEF = $\frac{1}{12}$ area of ΔPQR

$= \frac{1}{12} . \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$

$RS = \frac{1}{2}\sqrt{2(3)+2(1)-1} = \frac{\sqrt{7}}{2}$

$OE = \frac{1}{3} PE = \frac{1}{3} . \frac{1}{2}\sqrt{2(1)^2 + 2(1)^2 - 3} = \frac{1}{6}$

Q.8 (B,C)



$\tan\frac{x}{2} + \tan\frac{z}{2} = \frac{2y}{x+y+z}$

$\frac{\Delta}{S(S-x)} + \frac{\Delta}{S(S-z)} = \frac{2y}{2S}$

$\frac{\Delta}{S}\left(\frac{2S-(x+z)}{(S-x)(S-z)}\right) = \frac{y}{S}$

$\Rightarrow \frac{\Delta y}{S(S-x)(S-z)} = \frac{y}{S}$

$\Rightarrow \Delta^2 = (S-x)^2 (S-z)^2$

$\Rightarrow S(S-y) = (S-x)(S-z)$

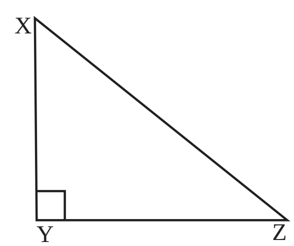
$\Rightarrow (x+y+z)(x+z-y) = (y+z-x)(x+y-z)$

$\Rightarrow (x+z)^2 - y^2 = y^2 - (z-x)^2$

$\Rightarrow (x+z)^2 + (x-z)^2 = 2y^2$

$\Rightarrow x^2 + z^2 = y^2 \Rightarrow \angle Y = \frac{\pi}{2}$

$\Rightarrow \angle Y = \angle X + \angle Z$



$\tan\frac{x}{2} = \frac{\Delta}{S(S-x)}$

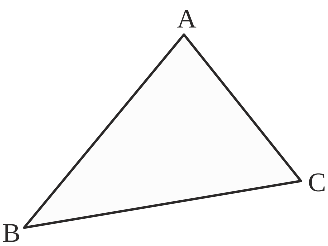
$\tan\frac{x}{2} = \frac{\frac{1}{2}xz}{\frac{(y+z)^2 - x^2}{4}}$

$$\tan\frac{x}{2} = \frac{2xz}{y^2+z^2+2yz-x^2}$$

$$\tan\frac{x}{2} = \frac{2xz}{2z^2+2yz}$$ (using $y^2=x^2+z^2$)

$$\tan\frac{x}{2} = \frac{x}{y+2}$$

Q.14 (2)



Given $c=\sqrt{23}$; a = 3; b = 4

$$\cot A = \frac{\cos A}{\sin A} = \frac{b^2+c^2-a^2}{2bc\sin A}$$

$$= \frac{b^2+c^2-a^2}{2.2\Delta}\left\{\Delta = \frac{1}{2}bc\sin A\right\}$$

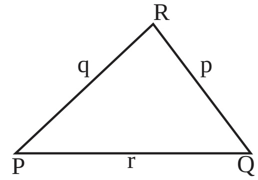
$$\cot A = \frac{b^2+c^2-a^2}{4\Delta}$$

Similarly, $\cot B = \frac{a^2+c^2-b^2}{4\Delta}$ & $\cot C = \frac{a^2+b^2-c^2}{4\Delta}$

$$\therefore \frac{\cot A+\cot C}{\cot B} = \frac{b^2+c^2-a^2+a^2+b^2-c^2}{a^2+c^2-b^2}$$

$$= \frac{2b^2}{a^2+c^2-b^2} = \frac{32}{16} = 2$$

Q.15 (A,B)



(A) $\text{Cos}P = \frac{q^2+r^2-p^2}{2qr} = \frac{q^2+r^2}{2qr} - \frac{p^2}{2qr} \geq 1 - \frac{p^2}{2qr}$

(as $p^2+q^2 \geq 2qr$(AM≥GM), so (A) is correct)

(B) (p+q)cosR≥(q–r)cos P+(p–r)cosQ

⇒ (pcosR+rcosP)+(qcosR+rcosQ)≥qcosP+pcosQ

⇒ q+p≥r

So (B) is correct

(C) $\frac{q+r}{p} = \frac{\sin Q+\sin R}{\sin P} \geq \frac{2\sqrt{\sin Q\times\sin R}}{\sin P}$

So (C) is incorrect

(D) $\cos Q > \frac{p}{r} \Rightarrow$ sinR cosQ>sinP

⇒ sinP+sin(R–Q)>2sinP

⇒ sin(R–Q)>sinP

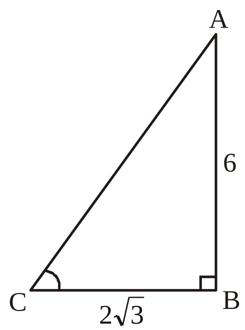
need not necessarily hold true if R<Q

Hence (A), (B)

Heights and Distance

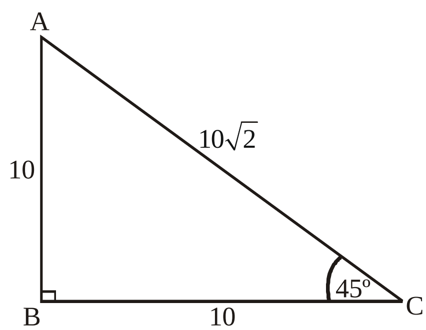
EXERCISES

Q.1 **(1)**



$\tan\theta = \dfrac{6}{2\sqrt{3}}$ $\Rightarrow$ $\theta = 60^o$

Q.2 (3)



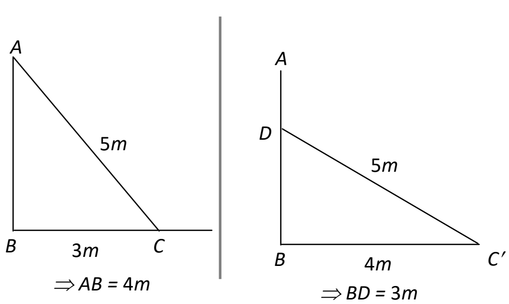
Total height of true $= AB + AC = 10(\sqrt{2}+1)$

Q.3 (2)

Required distance $= 60\cot 15^o = 60\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

Q.4 (1)

From first case, From second case,

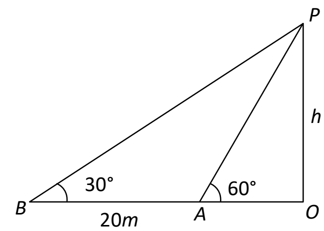


$\therefore AD = 4 - 3 = 1\text{m}.$

Q.5 (3)

$OA = h\cot 60^o, OB = h\cot 30^o$

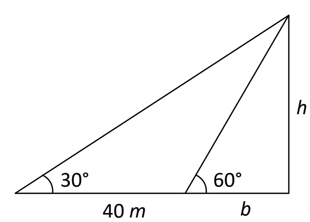
$OB - OA = 20 = h(\cot 30^o - \cot 60^o)$



$$h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

Q.6 (1)

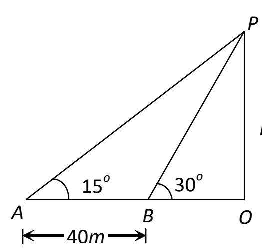
$b = h\cot 60^o, b + 40 = h\cot 30^o$



$$\Rightarrow \frac{b}{b+40} = \frac{\cot 60^o}{\cot 30^o} \Rightarrow b = 20\text{m}$$

Q.7 (2)

Let *h* be the height of pillar



$OB = h\cot 30^o$ and $OA = h\cot 15^o$

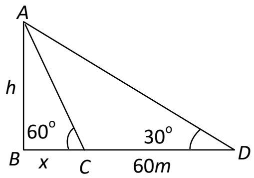
$\Rightarrow AB = OA - OB = h(\cot 15^o - \cot 30^o)$

$$\Rightarrow h = \frac{40}{\cot 15^o - \cot 30^o} = 20\ \textit{metre}.$$

Q.8 (4)

$\tan 30^o = \dfrac{h}{x+60}, \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+60}$

$x + 60 = \sqrt{3}h, \ x = \sqrt{3}h - 60$

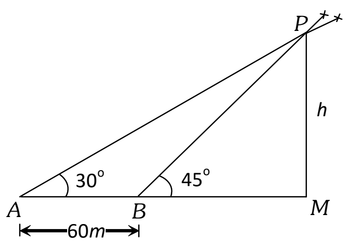


$$\tan 60^o = \frac{h}{x}\,,\ x = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h - 60 = \frac{h}{\sqrt{3}} \Rightarrow 3h - 60\sqrt{3} = h$$

$$\Rightarrow h = \frac{60\sqrt{3}}{2} = 30\sqrt{3} = 51.96 \approx 52m$$

Q.9 (4)

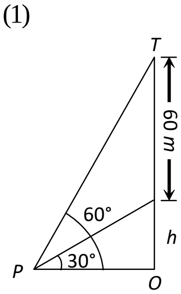


$$\because AB = AM - BM \Rightarrow \frac{AB}{h} = \frac{AM}{h} - \frac{BM}{h}$$

$$\frac{AB}{h} = \cot 30^o - \cot 45^o \Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1}$$

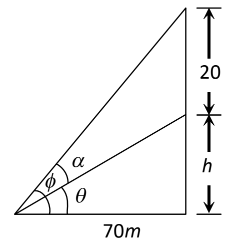
$$\Rightarrow h = 30(\sqrt{3}+1)\,m$$

Q.10 (1)



$$(60+h)\cot 60^o = h\cot 30^o \Rightarrow h = 30m$$

Q.11 (3)



$$\tan\alpha = \tan(\varphi - \theta)$$

$$\tan\alpha = \frac{1}{6} = \frac{\frac{20+h}{70} - \frac{h}{70}}{1 + \frac{(20+h)h}{(70)^2}}$$

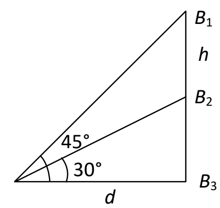
$$\Rightarrow (70)^2 + 20h + h^2 = (6)(70)(20)$$

$$\Rightarrow h^2 + 20h + 70(70-120) = 0$$

$$\Rightarrow h^2 + 20h - (50)(70) = 0$$

$$\Rightarrow h = \frac{-20 \pm \sqrt{400 + (4)(50)(70)}}{2} = 50m$$

Q.12 (2)



$$B_1B_2 = h = (d\tan 45^o - d\tan 30^o)$$

Time taken = 10 *min*

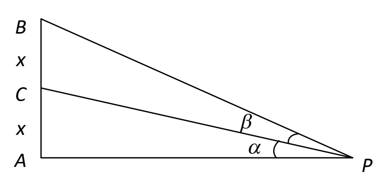
$$\text{Rate} = \ 4 = \frac{d}{10}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$$

$$\Rightarrow\ d = \frac{40\sqrt{3}}{\sqrt{3}-1} = 20(3+\sqrt{3})\,m\,.$$

Q.13 (2)

Let $AC = x = CB, AP = 3AB = 6x$. Let $\angle CPA = \alpha$

In $\Delta ACP, \tan\alpha = \frac{x}{6x} = \frac{1}{6}$



In ΔABP , $\tan(\alpha+\beta) = \frac{2x}{6x} = \frac{1}{3}$

Now $\tan\beta = \tan\{(\alpha+\beta)-\alpha\} = \dfrac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha}$

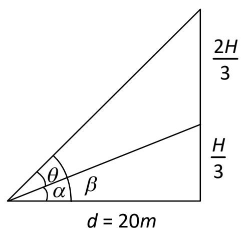
$$= \frac{\frac{1}{3}-\frac{1}{6}}{1+\frac{1}{3}.\frac{1}{6}} = \frac{1}{6}\times\frac{18}{19} = \frac{3}{19}$$

Q.14 (2)

$\dfrac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$

or $\dfrac{H}{3d} = \tan\alpha$ and $\dfrac{H}{d} = \tan\beta$

$$\tan(\beta-\alpha) = \frac{1}{2} = \frac{\frac{H}{d}-\frac{H}{3d}}{1+\frac{H^2}{3d^2}}$$



$$\Rightarrow 1+\frac{H^2}{3d^2} = \frac{4H}{3d}$$

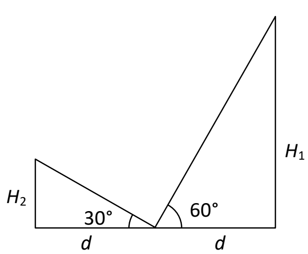
$$\Rightarrow H^2 - 4dH + 3d^2 = 0$$

$$\Rightarrow H^2 - 80H + 3(400) = 0$$

$$\Rightarrow H = 20 \text{ or } 60\text{m}$$

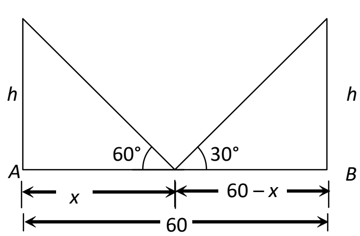
Q.15 (3)

$H_1 = d\tan 60^o$, $H_2 = d\tan 30^o$



$$\frac{H_1}{H_2} = \frac{\tan 60^o}{\tan 30^o} = \frac{3}{1}$$

Q.16 (1)



$\tan 60^\circ = \dfrac{h}{x} \Rightarrow \dfrac{\sqrt{3}}{1} = \dfrac{h}{x} \Rightarrow h = \sqrt{3}x$ .....(i)

$\tan 30^\circ = \dfrac{h}{60-x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{60-x} \Rightarrow 60-x = \sqrt{3}h$

.....(ii)

From equation (i) and (ii), $60-x = \sqrt{3}(\sqrt{3}x)$

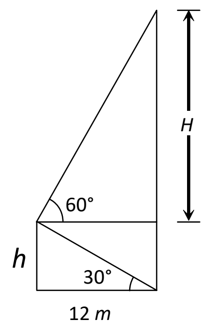
$$\frac{60}{4} = x \Rightarrow x = 15$$

Then $h = \sqrt{3}x \Rightarrow h = 15\sqrt{3}$ *metre.*

Q.17 (2)

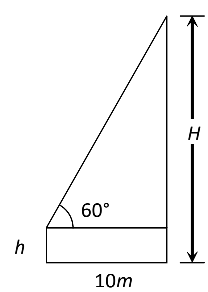
$h = 12\tan 30^o = \dfrac{12}{\sqrt{3}}$ and $H = 12\tan 60^o + \dfrac{12}{\sqrt{3}}$

$$12\sqrt{3} + \frac{12}{\sqrt{3}} = 16\sqrt{3}\text{m}$$



Q.18 (1)

$H = (10\tan 60^o + 1.5) = (10\sqrt{3} + 1.5)\text{m}$



Q.19 (1)

$64\cot\theta = d$

Also $(100-64)\tan\theta = d$

or $(64)(36) = d^2$

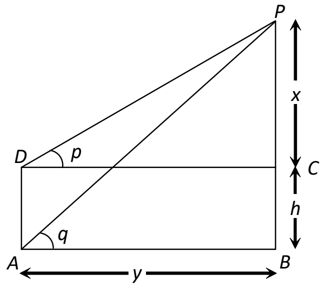
$\therefore d = 8\times 6 = 48$ m.

Q.20 (1)

Trick: From $H = l\tan\alpha.\tan\beta$, the height of tower is

$h\tan 30^o \cot 60^o$ *or* $\dfrac{h}{3}$

Q.21 (2)



Let *AD* be the building of height *h* and *BP* be the hill

then $\tan q = \dfrac{h+x}{y}$ and $\tan p = \dfrac{x}{y}$

$\Rightarrow \tan q = \dfrac{h+x}{x\cot p}$

$\Rightarrow x\cot p = (h+x)\cot q$

$\Rightarrow x = \dfrac{h\cot q}{\cot p - \cot q}$

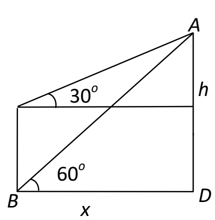
$\Rightarrow h + x = \dfrac{h\cot p}{\cot p - \cot q}$

Q.22 (1)

Total distance from temple $= \sqrt{x^2 + (240)^2}$ where

$x = \dfrac{h}{\tan 60^o} = \dfrac{h}{\sqrt{3}}$

So distance $= \sqrt{\dfrac{h^2}{3} + (240)^2}$



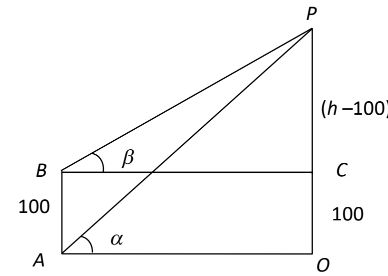
but $\dfrac{h}{\sqrt{\dfrac{h^2}{3} + (240)^2}} = \dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{h^2}{\dfrac{h^2}{3} + (240)^2} = \dfrac{1}{3}$

After solving, $h = 60\sqrt{6}$ m.

Q.23 (3)

If OP = h, then CP = h – 100

Now equate the values of OA and BC.



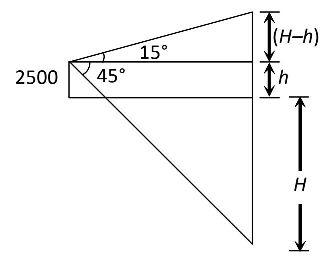
$h\cot\alpha = (h-100)\cot\beta$

$\therefore h = \dfrac{100\cot\beta}{\cot\beta - \cot\alpha}$

Q.24 (1)

$(H-h)\cot 15^o = (H+h)\cot 45^o$

or $H = \dfrac{h(\cot 15^o + 1)}{(\cot 15^o - 1)}$



Since $h = 2500$ and substitute

$\cot 15^o = 2 + \sqrt{3}$,we get, $H = 2500\sqrt{3}$

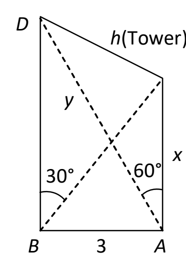
Q.25 (4)

From Δ CDA, $x = h\cot 60^o = \dfrac{h}{\sqrt{3}}$

From ΔCDB, y=h cot30°= $\sqrt{3}$h

From ΔABC, by Pythagoras theorem

$x^2 + 3^2 = y^2$



$\Rightarrow \left(\dfrac{h}{\sqrt{3}}\right)^2 + 3^2 = (\sqrt{3}h)^2 \Rightarrow h = \dfrac{3\sqrt{6}}{4}$ km.

Q.26 (3)

In

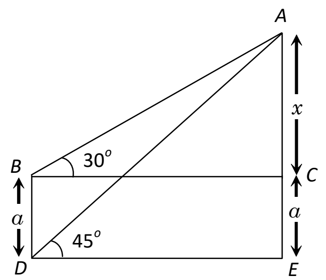
$\Delta ABC,\ \tan 30^o = \frac{AC}{BC}$ *or* $\frac{1}{\sqrt{3}} = \frac{x}{BC}$, where $AC = x$

or $BC = x\sqrt{3}$ and in

ΔADE, $\tan 45^o = \frac{a+x}{DE}$

or $1 = \frac{a+x}{x\sqrt{3}}$ or $x\sqrt{3} = a + x$, $x(\sqrt{3}-1) = a$

or $x = \frac{a}{\sqrt{3}-1}$.

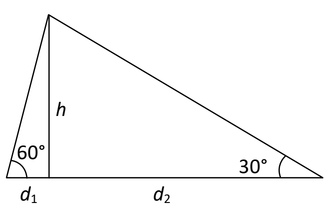


Therefore height of the tower, $a + x = a + \frac{a}{\sqrt{3}-1}$

$= a\left[\frac{\sqrt{3}-1+1}{\sqrt{3}-1}\right] = \frac{a\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{a(3+\sqrt{3})}{2}$.

Q.27 (2)

$d_2 = h\cot 30^o = 500\sqrt{3}, d_1 = \frac{500}{\sqrt{3}}$

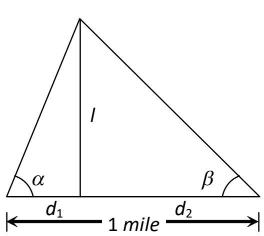


Diameter $D = 500\sqrt{3} + \frac{500}{3}\sqrt{3} = \frac{2000}{\sqrt{3}}$ m

Q.28 (4)

$d_1 = h\cot\alpha$ and $d_2 = h\cot\beta$

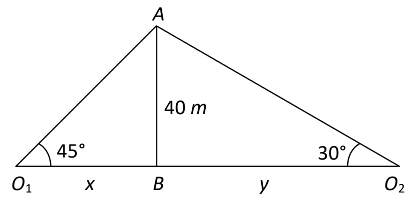
$d_1 + d_2 = 1$ *mile* $= h(\cot\alpha + \cot\beta)$



$\Rightarrow h = \frac{1}{(\cot\alpha + \cot\beta)} = \frac{\tan\alpha . \tan\beta}{\tan\alpha + \tan\beta}$

Q.29 (4)

From $\Delta\ O_1AB$, $\tan 45^o = \frac{40}{x} \Rightarrow x = 40m$



From ΔAO_2B, $\cot 30^o = \frac{y}{40}$

$\Rightarrow\ y = 40\ \cot\ 30^o = 40\sqrt{3}$

Distance between the men = $40 + 40\sqrt{3} = 109.28$ *m*.

Q.30 (4)

Let the two roads intersect at A. If the bus and the car are at B and C on the two roads respectively, then

c ,= AB = 2km, b = AC = 3 km. The distance between the two vehicles = BC = a km

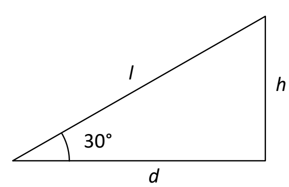
Now $\cos A = \cos 60^o = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \frac{1}{2} = \frac{3^2 + 2^2 - a^2}{2.3.2} \Rightarrow a = \sqrt{7}$ km.

JEE-MAIN OBJECTIVE QUESTIONS

Q.1 (3)

$H = 20 = l + h, l = \frac{d}{\cos 30^o}, h = d\tan 30^o$



$\therefore d = \frac{20}{(\sec 30^o + \tan 30^o)} = \frac{20}{\sqrt{3}}$

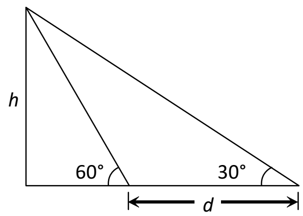
and hence $h = d\tan 30^o = \frac{20}{3}$m

Q.2 (1)

$\tan 30^o = h \Rightarrow h = 1/\sqrt{3}$ km

Q.3 (3)

$d = h\cot 30^o - h\cot 60^o$ *and time* $= 3$ min.



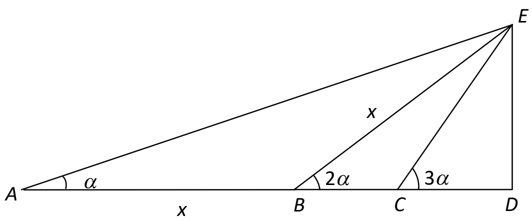
$\therefore$ Speed $= \dfrac{h(\cot 30^o - \cot 60^o)}{3}$ per minute

It will travel distance $h\cot 60^o$ in

$\dfrac{h\cot 60^o \times 3}{h(\cot 30^o - \cot 60^o)} = 1.5$ minute

Q.4 (2)

From sine rule,



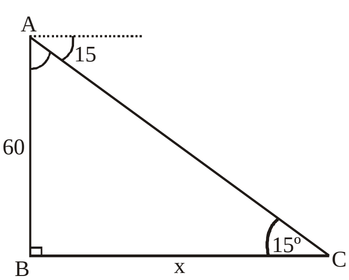
$\Rightarrow \dfrac{BE}{\sin(180^o - 3\alpha)} = \dfrac{BC}{\sin\alpha}$

$\Rightarrow \dfrac{AB}{\sin 3\alpha} = \dfrac{BC}{\sin\alpha}$ (Since $BE = AB$)

$\Rightarrow \dfrac{AB}{BC} = \dfrac{\sin 3\alpha}{\sin\alpha} = 3 - 4\sin^2\alpha$

$= 3 - 2(1-\cos 2\alpha) = 1 + 2\cos 2\alpha.$

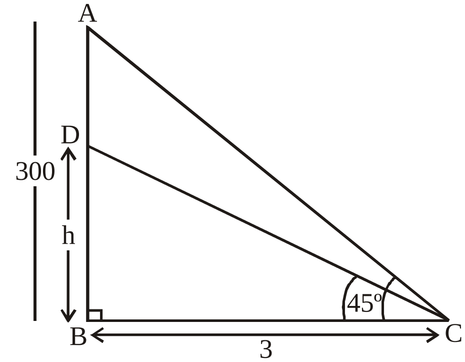
Q.5 **(2)**



If ΔABC $\dfrac{x}{60} = \cot 15^o$

$\Rightarrow x = (2+\sqrt{3})\,60 = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}.60$ metres

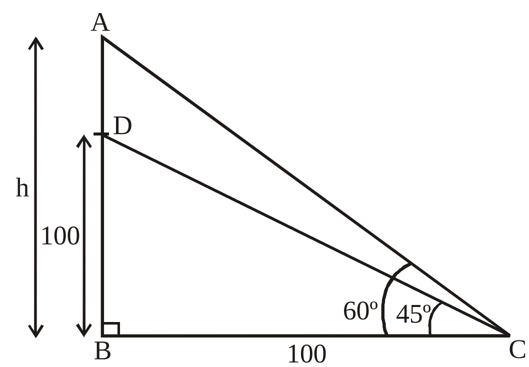
Q.6 **(1)**



$BC = 300\cot 60^o = \dfrac{100}{\sqrt{3}}$

If ΔBCD $\boxed{h = \dfrac{300}{\sqrt{3}}} = 100\sqrt{3}$

Q.7 **(3)**

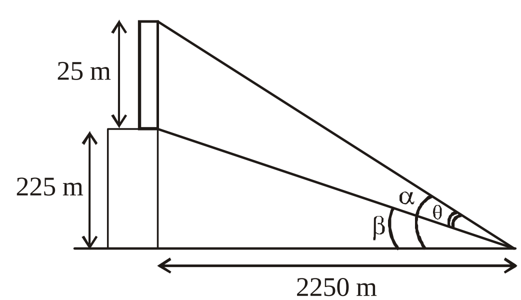


$\dfrac{h}{100} = \tan 60^o \quad \Rightarrow h = 100\sqrt{3}$

Height increased by $(100\sqrt{3} - 100)$

$= 100(\sqrt{3} - 1)$

Q.8 **(2)**



$\tan\alpha = \dfrac{250}{2250} = \dfrac{1}{9}$

$\tan\beta = \dfrac{225}{2250} = \dfrac{1}{10}$

$\theta = \alpha - \beta$

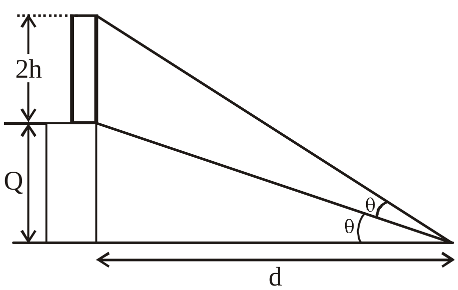
$$\tan\theta = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\,\tan\beta}$$

$$= \frac{\frac{1}{9} - \frac{1}{10}}{1 + \frac{1}{90}}$$

$$\tan\theta = \frac{1}{91}$$

$$\Rightarrow \boxed{\tan\theta = \frac{1}{91}}$$

Q.9 **(3)**



$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

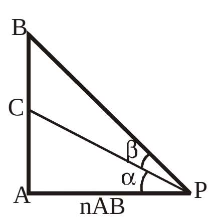
$$\tan 2\theta = \frac{3b}{d}$$

$$\tan\theta = \frac{h}{d}$$

$$\Rightarrow \frac{3h}{d} = \frac{\frac{2h}{d}}{1 - \frac{h^2}{d^2}} \Rightarrow \quad 1 - \left(\frac{h}{d}\right)^2 = \frac{2}{3}$$

$$\Rightarrow \boxed{\frac{1}{\sqrt{3}} = \frac{h}{d}}$$

Q.10 (A)



In ΔACP

$$\tan\alpha = \frac{AC}{AP} = \frac{AB/2}{nAP} = \frac{1}{2n}$$

In ΔABP

$$\tan(\alpha + \beta) = \frac{AB}{AP} = \frac{AB}{nAB} = \frac{1}{n}$$

$$\tan(\alpha + \beta) = \frac{1}{n}$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{n}$$

$$\frac{\frac{1}{2n} + \tan\beta}{1 - \frac{1}{2n}\tan\beta} = \frac{1}{4}$$

$$\frac{\frac{1 + 2n\tan\beta}{2n}}{\frac{2n - \tan\beta}{2n}} = \frac{1}{n}$$

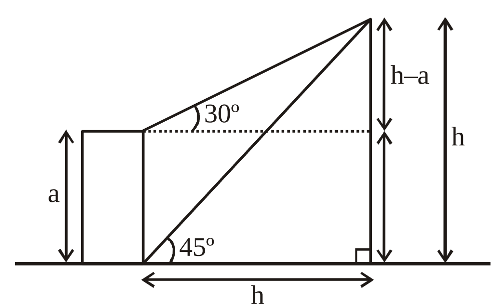
$n(1 + 2n\tan\beta) = 2n - \tan\beta$

$n + 2n^2\tan\beta = 2n - \tan\beta$

$\tan\beta\,(2n^2 + 1) = 2n - n$

$$\tan\beta = \frac{n}{2n^2 + 1}$$

Q.11 **(3)**



$$\frac{1}{\sqrt{3}} = \frac{h - a}{h}$$

$h = h\sqrt{3} - a\sqrt{3} \quad \Rightarrow \quad a\sqrt{3} - h(\sqrt{3} - 1)$

$$\Rightarrow h = \frac{a\sqrt{3}}{(\sqrt{3} - 1)} = a(3 - \sqrt{3})$$

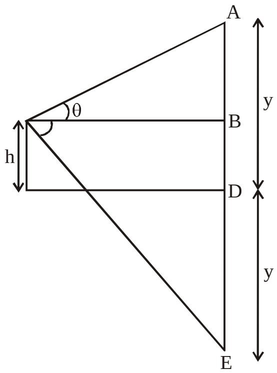
$$\boxed{h = \frac{a\sqrt{3}}{(\sqrt{3} - 1)}}$$

Q.12 **(2)**

In Δ ABC at BCE

$(y-h)\cot\theta = (y+h)\cot\phi$

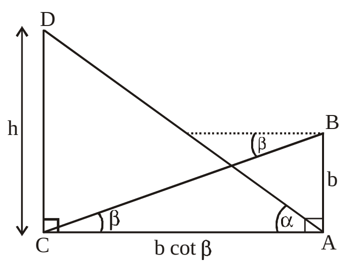
$y(\cot\theta - \cot\phi) = (\cot\phi + \cot\theta)h$



$$\Rightarrow y = \frac{(\cot\phi + \cot\theta)h}{\cot\theta - \cot\phi}$$

$$y = \frac{\sin(\theta+\phi)}{\sin(\phi-\theta)}h$$

Q.13 **(1)**



In Δ ABC

AC = b cot β

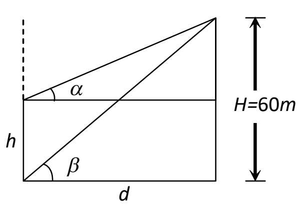
hΔ ACD

h = b cot β tan α.

Q.14 (4)

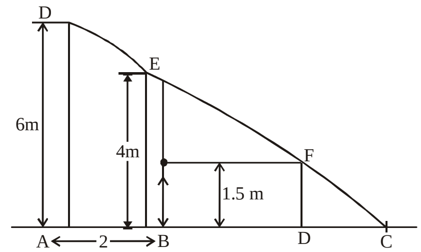
$H = d\tan\beta$ and $H - h = d\tan\alpha$

$$\Rightarrow \frac{60}{60-h} = \frac{\tan\beta}{\tan\alpha} \Rightarrow -h = \frac{60\tan\alpha - 60\tan\beta}{\tan\beta}$$



$$\Rightarrow h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\dfrac{\sin\beta}{\cos\beta}} \Rightarrow x = \cos\alpha\sin\beta$$

Q.15 **(1)**



$$BD \Rightarrow \frac{BE}{BC} = \frac{FD}{CD}$$

$$\frac{AD}{AC} = \frac{BE}{BC}$$

$$\frac{4}{2_1} = \frac{1.5}{4-y}$$

$$\frac{6}{2+x} = \frac{y}{x}$$

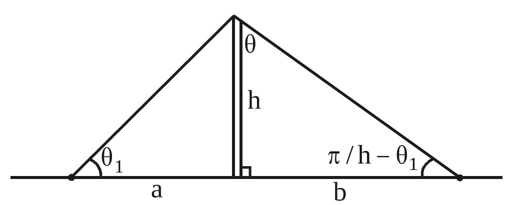
$$4 - y = 1.5 \Rightarrow y = 2.5 = \boxed{\frac{5}{2}}$$

$6x = 8 + 4x$

$8x = 8$

$n = 4m$

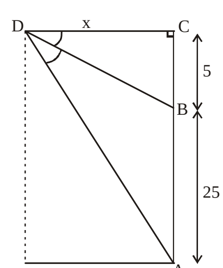
Q.16 **(4)**



$$\tan\theta_1 \tan\left(\frac{\pi}{2} - \theta_1\right) = 1$$

$$\frac{h}{a} - \frac{h}{b} = 1 \Rightarrow h = \sqrt{ab}$$

Q.17 **(2)**



In ΔBCD In ΔACD

$$\tan\alpha = \frac{5}{x} \;;\; \tan 2\alpha = \frac{30}{x}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{30}{x}$$

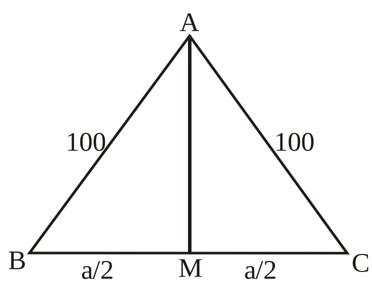
$$\frac{2\times\frac{5}{x}}{1-\frac{25}{x^2}} = \frac{30}{x}$$

$$\frac{10}{x^2}.\frac{x^2}{(x^2-25)} = \frac{30}{x} \Rightarrow x^2 = 3c^2 - 75$$

$$75 = 2x^2$$

$$x = 5\sqrt{\frac{3}{2}}$$

Q.18 **(2)**



$$AM = \sqrt{2(100)^2 + 2(100)^2 - a^2}$$

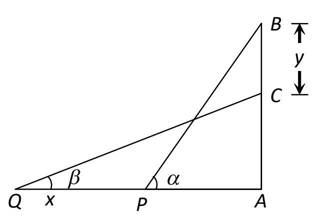
given $\frac{AM}{h} = \frac{16}{5}$ ......(1)

$\frac{h}{a/2} = \frac{5}{12}$ ......(2)

Q.19 (1)

PB = QC = l (Length of ladder)

$\Rightarrow$ PA = l cos α, QA = l cos β



$\Rightarrow$ AC = l sin β, AB = l sin α

$\Rightarrow$ CB = AB – AC = l(sin α – sin β)

$\Rightarrow$ y = l(sin α – sin β)

and QP = x = AQ – AP = l , (cos β – cos α)

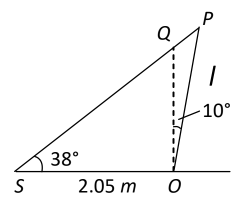
$\Rightarrow$

$$\frac{CB}{QP} = \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{y}{x} = \frac{2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow \frac{y}{x} = \cot\left(\frac{\alpha+\beta}{2}\right) \Rightarrow x = y\tan\left(\frac{\alpha+\beta}{2}\right)$$

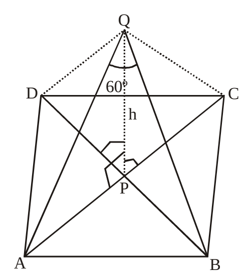
Q.20 (1)

$$\frac{\sin 38^o}{l} = \frac{\sin(SPO)}{2.05}$$



$$= \frac{\sin(180^o - 38^o - 90^o - 10^o)}{2.05} \Rightarrow l = \frac{2.05\sin 38^o}{\sin 42^o}$$

Q.21 **(2)**



$\angle$ AQB = 60° and AQ = BQ

$\Rightarrow$ ABQ is equilateral triangle

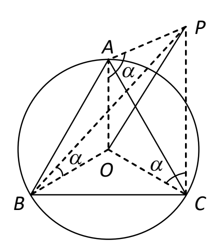
In ΔDPQ

$AQ^2 = h^2 + AP^2$

$Q^2 = h^2 + \frac{a^2}{2} \Rightarrow \boxed{2h^2 = a^2}$

Q.22 (4)

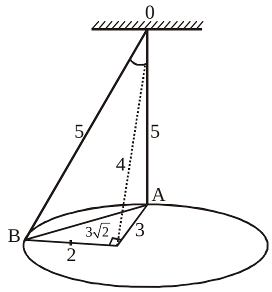
Since the tower makes equal angles at the vertices of the triangle, therefore foot of the tower is at the circumcentre.



From ΔOAP, we have $\tan\alpha = \frac{OP}{OA}$

$\Rightarrow$ OP = OA tan α OP = R tan α

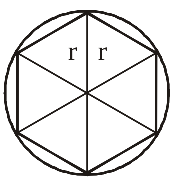
Q.23 **(3)**



$$\cos\theta = \frac{5^2+5^2-(3\sqrt{2})^2}{2(5)(5)} = \frac{25+25-18}{50}$$

$$\cos\theta = \frac{32}{50} = \frac{16}{25}$$

Q.24 **(4)**



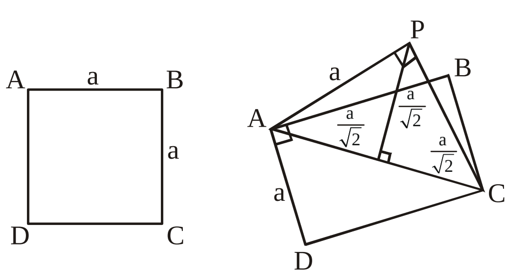
$$\tan\left(\frac{\pi}{3}\right) = \frac{h}{r}$$

Area of hexagen = $\frac{3\sqrt{3}}{2}r^2$

Area of circle = πr^2 = A

$$\Rightarrow \frac{3\sqrt{3}}{2}\cdot\left(\frac{A}{\pi}\right) = \frac{3\sqrt{3}\,\pi}{2\pi}\ m^2$$

Q.25 (3)



$\Rightarrow AP = AD = PD = a$

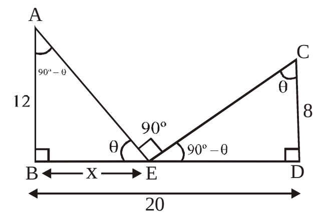
$\Rightarrow$ angle subtended

by aside = $\frac{\pi}{3}$

($\because$ equilateral Δu)

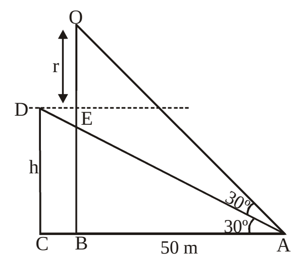
KVPY
PREVIOUS YEAR'S

Q.1 (A)



$$\frac{12}{x} = \frac{20-x}{8} \Rightarrow x^2-20x+96=0 \quad \Rightarrow x = 8,\ 12$$

Q.2 (C)



DE = BC = r

$$\tan 30° = \frac{h}{50}$$

$$\boxed{h = \frac{50}{\sqrt{3}}}$$

$$\tan 60° = \frac{h+r}{50-r}$$

$$\sqrt{3}(50-r) = h+r$$

$$\sqrt{3}(50-r) = \frac{50}{\sqrt{3}}+r$$

$$3(50-r) = 50+\sqrt{3}\,r$$

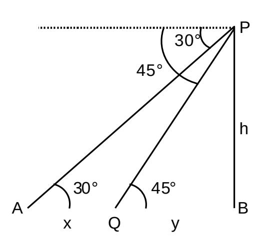
$$100 = \left(3+\sqrt{3}\right)r$$

$$r = \frac{100}{3+\sqrt{3}}$$

$$r = \frac{100\left(3-\sqrt{3}\right)}{6} = 50\left(1-\frac{1}{\sqrt{3}}\right)$$

JEE MAINS
PREVIOUS YEAR'S

Q.1 (3)



In ΔABP

$$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}}$$

$x + y = \sqrt{3}h$ ...(i)

In ΔQBP

$$\frac{h}{y} = \tan 45^o = 1$$

$h = y$ ...(ii)

$x + y = \sqrt{3}\, y$

$x = (\sqrt{3} - 1)y$

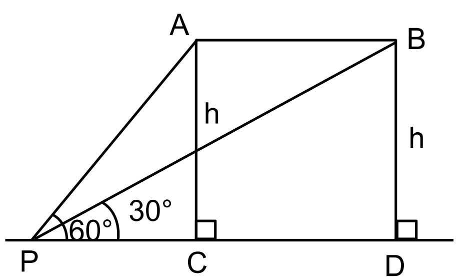
Let speed is v

$\frac{x}{v} = 20 \Rightarrow x = 20\, v$

$\therefore\ 20\, v = (\sqrt{3} - 1)\, y$

Time to cover y distance $= \frac{y}{v} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3} + 1)$ sec

Q.2 (1)



$v = 432 \times \frac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC}$ $\Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

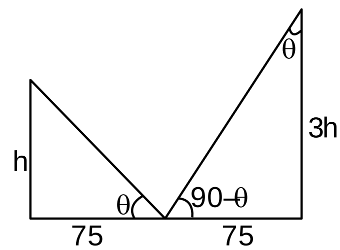
$\tan 30° = \frac{h}{PC}$ $\Rightarrow PC = \sqrt{3}\, h$

PD = PC + CD

$\sqrt{3}\, h = \frac{h}{\sqrt{3}} + 2400$ $\Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

Q.3 (1)



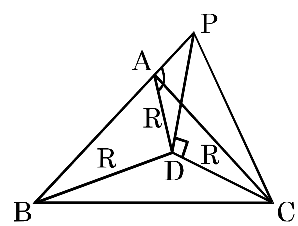
$\tan\theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$ m

Q.4 (2)

Let PD = h, R = 2



As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC

$$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$$

$$h = R\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

Q.5 (2)

Q.6 (2)

Q.7 (1)

Q.8 **(2)**

Quadratic Equations

EXERCISES

ELEMENTRY

Q.1 (3)

Given equation is

$(p - q)x^2 + (q - r)x + (r - p) = 0$

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p(p-q)}}{2(p-q)}$$

$$\Rightarrow x = \frac{(r-q) \pm \sqrt{(q+r-2p)}}{2(p-q)} \Rightarrow x = \frac{r-p}{p-q}, 1$$

Q.2 (3)

We have $4ax^2 + 3bx + 2c = 0$ Let roots are α and β

Let $D = B^2 - 4AC = 9b^2 - 4(4a)(2c) = 9b^2 - 32ac$

Given that, $(a + b + c) = 0 \Rightarrow b = -(a + c)$

Putting this value, we get

$= 9(a + c)^2 - 32ac = 9(a - c)^2 + 4ac$

Hence roots are real.

Q.3 (4)

Given equation

$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

Let $A = 2(a^2 + b^2)$, $B = 2(a + b)$ and $C = 1$

$B^2 - 4AC = 4(a^2 + b^2 + 2ab) - 4.2(a^2 + b^2)1$

$\Rightarrow B^2 - 4AC = -4(a - b)^2 < 0$

Thus given equation has imaginary roots.

Q.4 (2)

The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0$, $b > 0$

Now if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \ (i = \sqrt{-1})$$

Which are imaginary and have negative real part

$(\because b > 0)$

$\therefore$ In each case, the roots have negative real part.

Q.5 (1)

Here $(b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$

Therefore the roots are rational.

Q.6 (3)

The quadratic is $(k + 11)x^2 - (k + 3)x + 1 = 0$

Accordingly, $(k + 3)^2 - 4(k + 11)(1) = 0$

$\Rightarrow k = -7, 5$

Q.7 (3)

From options put $k = 3 \Rightarrow x^2 + 8x + 7 = 0$

$\Rightarrow (x + 1)(x + 7) = 0 \Rightarrow x = -1, -7$

means for k = 3 roots are negative.

Q.8 (1)

Given equation $(1 + 2k)x^2 + (1 - 2k)x + (1 - 2k) = 0$

If equation is a perfect square then root are equal

i.e., $(1 - 2k)^2 - 4(1 + 2k)(1 - 2k) = 0$

i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values = 2.

Q.9 (2)

Let first root = α and second root = $\frac{1}{\alpha}$

Then $\alpha, \frac{1}{\alpha} = \frac{k}{5} \Rightarrow k = 5$

Q.10 (1)

Given equation $4x^2 + 3x + 7 = 0$, therefore

$\alpha + \beta = -\frac{3}{4}$ and $\alpha\beta = \frac{7}{4}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3/7}{7/4} = \frac{-3}{4} \times \frac{4}{7} = -\frac{3}{7}$

Q.11 (1)

Here $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

If roots are $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ then sum of roots are

$$= \left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = (\alpha + \beta) + \frac{(\alpha+\beta)}{\alpha\beta} = \frac{b}{ac}(a + c)$$

and product $= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = 2 + \frac{c}{a} + \frac{a}{c}$$

$$= \frac{2ac + c^2 + a^2}{ac} = \frac{(a+c)^2}{ac}$$

Hence required equation is given by

$$x^2 + \frac{b}{ac}(a + c)x + \frac{(a+c)^2}{ac} = 0$$

$\Rightarrow acx^2 + (a + c)bx + (a + c)^2 = 0$

Trick : Let a = 1 , b = – 3, c = 2, then α = 1, β = 2b = –3, c = 2, then α= 1, β = 2

$$\therefore\ \alpha+\frac{1}{\beta}=\frac{3}{2}\ \text{and}\ \beta+\frac{1}{\alpha}=3$$

Therefore, required equation must be

$(x – 3)(2x – 3) = 0$ i.e. $2x^2 – 9x + 9 = 0$

Here (1) gives this equation on putting

a = 1, b = – 3, c = 2

Q.12 (4)

$$\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$$

$$\text{and } \alpha^2+\beta^2=\frac{(b^2-2ac)}{a^2}$$

$$\text{Now } \frac{\alpha}{a\beta+b}+\frac{\beta}{a\alpha+b}=\frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$$

$$=\frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+ab(\alpha+\beta)+b^2}=\frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$$

$$=\frac{b^2-ac-b^2}{a^2c-ab^2+ab^2}=\frac{-ac}{a^2c}=-\frac{2}{a}$$

Q.13 (3)

Let α and β be two roots of $ax^2 + bx + c = 0$

$$\text{Then } \alpha+\beta=-\frac{b}{a}\ \text{and}\ \alpha\beta=\frac{c}{a}$$

$$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{b^2}{a^2}-2\frac{c}{a}$$

So under condition $\alpha + \beta = \alpha^2 + \beta^2$ $\alpha + \beta = a^2 + \beta^2$

$$\Rightarrow -\frac{b}{a}=\frac{b^2-2ac}{a^2}\Rightarrow b(a+b)=2ac$$

Q.14 (2)

α, β be the roots of $x^2 – 2x + 3 = 0$, then α + β = 2 and α β = 3. Now required equation whose roots are

$$\frac{1}{\alpha^2},\frac{1}{\beta^2}\ \text{is}$$

$$x^2-\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x+\frac{1}{\alpha^2\beta^2}=0$$

$$\Rightarrow x^2-\left(-\frac{2}{9}\right)x+\frac{1}{9}=0\Rightarrow 9x^2+2x+1=0$$

Q.15 (3)

According to condition

$$\frac{2m-1}{m}=-1\Rightarrow 3m=1\Rightarrow m=\frac{1}{3}$$

Q.16 (2)

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 – 3\alpha\beta(\alpha + \beta)$

$= (4)^3 – 3 \times 1(4) = 52$

Q.17 (1)

$$\text{Let roots are } \alpha, \beta \text{ so, } \frac{\alpha}{\beta}=\frac{2}{3}\Rightarrow\alpha=\frac{2\beta}{3}$$

$$\therefore\ \alpha+\beta=\frac{m}{12}$$

$$\Rightarrow \frac{2\beta}{3}+\beta=\frac{m}{12}\Rightarrow\frac{5\beta}{3}=\frac{m}{12}\quad .....(i)$$

$$\text{and } \alpha\beta=\frac{5}{12}\Rightarrow\frac{2\beta}{3}.\beta=\frac{5}{12}\Rightarrow\beta^2=\frac{5}{8}$$

$$\Rightarrow\beta=\sqrt{5/8}$$

Put the value of β in (i), $\frac{5}{3}.\sqrt{\frac{5}{8}}=\frac{m}{12}\Rightarrow m=5\sqrt{10}$.

Q.18 (2)

Given equation can be written as

$(6k + 2)x^2 + rx + 3k – 1 = 0$ .....(i)

and $2(6k + 2)x^2 + px + 2(3k – 1) = 0$ .....(ii)

Condition for common roots is

$$\frac{12k+4}{6k+2}=\frac{p}{r}=\frac{6k-2}{3k-1}=2 \text{ or } 2r-p=0$$

Q.19 (1)

Let α is the common root,

so $\alpha^2 + p\alpha + q = 0$ .....(i)

and $\alpha^2 + q\alpha + p = 0$ ....(ii)

from (i) – (ii),

$\Rightarrow (p – q)\alpha + (q – p) = 0 \Rightarrow \alpha = 1$

Put the value of α in (i), p + q + 1 = 0.

Q.20 (2)

Expressions are $x^2 – 11x + a$ and $x^2 – 14x + 2a$ will have a common factor, then

$$\Rightarrow\frac{x^2}{-22a+14a}=\frac{x}{a-2a}=\frac{1}{-14+11}$$

$$\Rightarrow\frac{x^2}{-8a}=\frac{x}{-a}=\frac{1}{-3}\Rightarrow x^2=\frac{8a}{3}\ \text{and}\ x=\frac{a}{3}$$

$$\Rightarrow\left(\frac{a}{3}\right)^2=\frac{8a}{3}\Rightarrow\frac{a^2}{9}=\frac{8a}{3}\Rightarrow a=0,24.$$

Trick : We can check by putting the values of *a* from the options.

Q.21 (2)

Given , $x + 2 > \sqrt{x+4} \Rightarrow (x+2)^2 > (x + 4)$

$\Rightarrow x + 4x + 4 > x + 4 \Rightarrow x^2 + 3x > 0$

$\Rightarrow x (x + 3) > 0 \Rightarrow x < -3$ or $x > 0 \Rightarrow x > 0$

Q.22 (2)

Case I: When $x + 2 \geq 0$ i.e. $x \geq -2$

Then given inequality becomes

$x^2 - (x + 2) + x > 0 \Rightarrow x^2 - 2 > 0 \Rightarrow |x| > \sqrt{2}$

$\Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

As $x \geq -2$, therefore, in this case the part of the solution set is $[-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

Case II: When $x + 2 \leq 0$ *i.e.* $x \leq -2$,

Then given inequality becomes $x^2 + (x + 2) + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0 \Rightarrow (x + 1)^2 + 1 > 0$, which is true for all real x

Hence, the part of the solution set in this case is $(-\infty, -2]$. Combining the two cases, the solution set is

$(-\infty, -2) \cup ([-2, -\sqrt{2}] \cup (\sqrt{2}, \infty) = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

Q.23 (3)

If α, β, γ are the roots of the equation.

$x^3 - px^2 + qx - r = 0$

$\therefore (\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} = \dfrac{p^2 + q}{pq - r}$

Given, p = 0, q = 4 , r = – 1

$\Rightarrow \dfrac{p^2 + q}{pq - r} = \dfrac{0+4}{0+1} = 4$

Q.24 (4)

We know that the roots of the equation

$ax^3 + bx^2 + cx + d = 0$ follows $\alpha\beta\gamma = -d/a$

Comparing above equation with given equation

we get d = 1, a = 1

So, $\alpha\beta\gamma = -1$ or $\alpha^3 \beta^3 \gamma^3 = -1$.

Q.25 (3)

Let $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4} \Rightarrow (y - 1)x^2 + 3(y+1)x + 4(y - 1) = 0$

For x is real $D \geq 0$

$\Rightarrow 9(y + 1)^2 - 16(y - 1)^2 \geq 0$ $9(y + 1)^2 - 16(y - 1)^2 \geq 0$

$\Rightarrow -7y^2 + 50y - 7 \geq 0 \Rightarrow 7y^2 - 50y + 7 \leq 0$

$\Rightarrow (y - 7)(7y - 1) \leq 0$

Now, the product of two factors is negative if one in –ve and one in +ve.

Case I : $(y - 7) \geq 0$ and $(7y - 1) \leq 0$

$\Rightarrow y \geq 7$ and $y \geq \dfrac{1}{7}$. But it is impossible

Case II : $(y - 7) \leq 0$ and $(7y - 1) \geq 0$

$\Rightarrow y \leq 7$ and $y \geq \dfrac{1}{7} \Rightarrow \dfrac{1}{7} \leq y \leq 7$

Hence maximum value is 7 and minimum value is $\dfrac{1}{7}$

Q.26 (1)

$x^2 - 3x + 3 = \left(x - \dfrac{3}{2}\right)^2 + \dfrac{3}{4}$

Therefore, smallest value is $\dfrac{3}{4}$, which lie in $\left(-3, \dfrac{3}{2}\right)$

Q.27 (4)

$x^2 - 3x + 2$ be factor of $x^4 - px^2 + q = 0$

Hence $(x^2 - 3x + 2) = 0 \Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 2, 1$ putting these values in given equation

so $4p - q - 16 = 0$ .....(i)

and $p - q - 1 = 0$ .....(ii)

Solving (i) and (ii), we get (p, q) = (5, 4)

Q.28 (4)

If the roots of the quadratic equation $ax^2 + bx + c = 0$ exceed a number k , then $ak^2 + bk + c > 0$ if $a > 0$, $b^2 - 4ac \geq 0$ and sum of the roots > 2k. Therefore, if the roots of $x^2 + x + a = 0$ exceed a number a , then $a^2 + a + a > 0$, $1 - 4a \geq 0$ and $-1 > 2a$

$\Rightarrow a(a + 2) > 0$, $a \leq \dfrac{1}{4}$ and $a < -\dfrac{1}{2}$

$\Rightarrow a > 0$ or $a < -2, a < \dfrac{1}{4}$ and $a < -\dfrac{1}{2}$

Hence a < – 2.

Q.29 (4)

Let

$f(x) = 4x^2 - 20px + (25p^2 + 15p - 66) = 0$ .....(i)

The roots of (i) are real if

$b^2 - 4ac = 400p^2 - 16(25p^2 + 15p - 66)$

$= 16(66 - 15p) \geq 0$

$\Rightarrow p \leq 22/5$ .....(ii)

Both roots of (i) are less than 2. Therefore f(2) > 0 and sum of roots < 4.

$\Rightarrow 4.2^2 - 20p.2 + (24p^2 + 15p - 66) > 0$ and $\dfrac{20p}{4} < 4$

$\Rightarrow p^2 - p - 2 > 0$ and $p < \dfrac{4}{5}$

$\Rightarrow (p + 1)(p - 2) > 0$ and $p < \dfrac{4}{5}$

$\Rightarrow p < -1$ or $p > 2$ and $p < \frac{4}{5} \Rightarrow p < -1$ .....(iii)

From (ii) and (iii), we get $p < -1$ i.e. $p \in (-\infty, -1)$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

check by options

x = 1 is root

Let other root = a

$\therefore$ Product of the roots = (1) (a) = $\frac{a-b}{b-c}$

$\Rightarrow$ roots are 1, $\frac{a-b}{b-c}$

Q.2 (1)

$D = b^2 - 4ac = 20d^2$

$\sqrt{D} = 2\sqrt{5}d$ here $\sqrt{5}$ is irrational

So roots are irrational.

Q.3 (1)

$D = b^2 - 4ac = b^2 - 4a(-4a - 2b)$

$= b^2 + 16a^2 + 8ab$

Since ab > 0

$\therefore$ D > 0

So equation has real roots.

Q.4 (3)

For integral roots, D of equation should be perfect sq.

$\because$ D = 4(1+n)

By observation, for $n \in N$, D should be perfect sq. of even integer.

So $D = 4(1+n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$

No. of values of n = 8.

Q.5 (2)

a > 0 & c < 0 is satisfied by (B) only

[$\because$ f(0) = 0 & a > 0]

Further in (B)

$-\frac{b}{2a} > 0 \Rightarrow b < 0$ [$\because$ a > 0].

Q.6 (2)

Here for D < 0 , entire graph will be above x-axis ($\because$ a > 0)

$\Rightarrow (k-1)^2 - 36 < 0$

$\Rightarrow (k-7)(k+5) < 0$

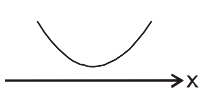
$\Rightarrow -5 < k < 7$

Q.7 (1)

Let $f(x) = ax^2 - bx + 1$

Given D < 0 & f(0) = 1 > 0

$\therefore$ possible graph is as shown



i.e. $f(x) > 0 \ \forall \ x \in R$

or f(–1) > 0

f(–1) = a + b + 1 > 0

Q.8 (3)

$x^2 + ax + b = 0 \ < \begin{matrix} a \\ b \end{matrix}$

a + b = – a

$\Rightarrow$ 2a + b = 0

and ab = b

ab – b = 0

b (a – 1) = 0

$\Rightarrow$ Either b = 0 or a = 1

But $b \neq 0$(given)

$\therefore$ a = 1

$\therefore$ b = – 2

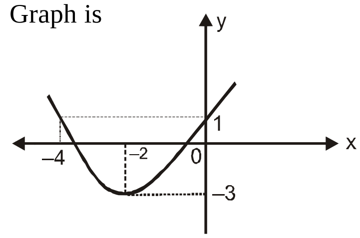
$\therefore$ $f(x) = x^2 + x - 2$

Least value occurs at $x = -\frac{1}{2}$

Least value = $\frac{1}{4} - \frac{1}{2} - 2 = -\frac{9}{4}$

Q.9 (3)

Graph is



From this graph all the four options can be checked.

Q.10 (2)

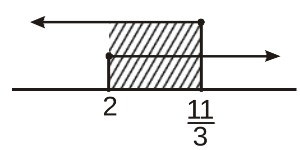
(2 – x) (x + 1) = p

(x – 2) (x + 1) + p = 0

$\Rightarrow x^2 - x - 2 + p = 0$

$\frac{c}{a} > 0 \Rightarrow p - 2 > 0$

& $D > 0 \Rightarrow 1 - 4(p-2) > 0 \Rightarrow p < \frac{9}{4}$



$\frac{-b}{2a} > 0, \ \frac{-1}{2(2-p)} > 0, \quad P \in (2, \infty)$

Taking intersection of all $p \in \left(2, \frac{9}{4}\right)$

Q.11 (3)

$x^2 + px + (1-p) = 0$

$(1-p)^2 + p(1-p) + (1-p) = 0$

$(1-p)[1 - p + p + 1] = 0 \Rightarrow p = 1$

Q.E. will be $\Rightarrow x^2 + x = 0 \Rightarrow x(x+1) = 0$

$\Rightarrow x = 0, -1$

Aliter

$\alpha + 1 - p = -p \Rightarrow \alpha = -1$

Satisfies

$1 - p + 1 - p = 0 \Rightarrow p = 1$

$\beta = 1 - p = 0 \Rightarrow \beta = 0$

Q.12 (3)

$a + b = -p$

$ab = q$

$g + d = -p$

$gd = -r$

$(a - g)(a - d) = a^2 - a(g + d) + gd$

$= a^2 + pa - r$

$= a(a + p) - r$

$= -ab - r$

$= -q - r$

$= -(q + r)$

Q.13 (1)

$|a - b| = 4 \Rightarrow (a - b)^2 = 16 \quad \Rightarrow (a + b)^2 - 4ab = 16$

$\Rightarrow 9 - 4ab = 16 \Rightarrow ab = -\frac{7}{4} \Rightarrow$ equation is $x^2 - 3x - \frac{7}{4} = 0$

Q.14 (1)

$C_1 : b^2 - 4ac \geqslant 0$,

$ax^2 + bx + c = 0$ real roots C_1 satisfied

C_2 : a, –b, c are same sign

$\alpha + \beta > 0 \Rightarrow \frac{-b}{a} > 0$

$\alpha\beta > 0 \Rightarrow \frac{c}{a} > 0$

C_2 satisfied

C_1 & C_2 are satisfied

Q.15 (2)

$x^2 + 2ax + b = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$D > 0, |\alpha - \beta| \leqslant 2m$

$4a^2 - 4b > 0$

$a^2 - b > 0$

$\Rightarrow b < a^2, \ \alpha + \beta = -2a, \ \alpha\beta = b$

$|\alpha - \beta|^2 \leqslant (2m)^2$

$(-2a)^2 - 4(b) \leqslant 4m^2$

$a^2 - b \leqslant m^2$

$b \geqslant a^2 - m^2$

$b \in [a^2 - m^2, a^2)$

Q.16 (3)

$ax^2 + bx + c = 0, \ \alpha + \beta = \frac{-b}{a}, \ \alpha\beta = \frac{c}{a}$

$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\frac{c}{a}\right]$$

$$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right] = \frac{-b}{a}\frac{(b^2 - 3ac)}{a^2} = \frac{3abc - b^3}{a^3}$$

Q.17 (1)

$x^2(6k + 2) + rx + (3k - 1) = 0$

$x^2(12k + 4) + px + 6k - 2 = 0$

For both roots common,

$$\frac{6k + 2}{12k + 4} = \frac{r}{p} = \frac{3k - 1}{2(3k - 1)} \Rightarrow \frac{r}{p} = \frac{1}{2}$$

$\Rightarrow 2r - p = 0$ Ans.

Q.18 (1)

$D_1 = 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$ img. root

$D_2 = 4p^2q^2 - 4p^2q^2 = 0$ equal, real roots

So no common roots.

Q.19 (2)

For $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ to be an identity

$p^2 - 3p + 2 = 0 \Rightarrow p = 1, 2$ ...(1)

$p^2 - 5p + 4 = 0 \Rightarrow p = 1, 4$ ...(2)

$p - p^2 = 0 \Rightarrow p = 0, 1$ ...(3)

For (1), (2) & (3) to hold simultaneously

$p = 1$.

Q.20 (4)

$x^2 + 9 < (x + 3)^2 < 8x + 25$

$x^2 + 9 < x^2 + 6x + 9 \Rightarrow \ x > 0$

& $(x + 3)^2 < 8x + 25$

$x^2 + 6x + 9 - 8x - 25 < 0$

$x^2 - 2x - 16 < 0$

$1 - \sqrt{17} < x < 1 + \sqrt{17}$ & $x > 0$

$\Rightarrow x \in (0, 1 + \sqrt{17})$

Integer x = 1, 2, 3, 4, 5

No. of integer are = 5

Q.21 (4)

$5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$ ...(i)

$$\frac{x + 2}{x - 1} < 4 \Rightarrow \frac{x + 2}{x - 1} - 4 < 0 \Rightarrow \frac{-3x + 6}{x - 1} < 0$$

$\Rightarrow \frac{x - 2}{x - 1} > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ ...(ii)

Taking intersection of (i) and (ii) $x \in (-\infty, 1) \cup (2, 3)$

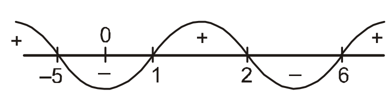
Q.22 (2)

$$\frac{x^2(x^2 - 3x + 2)}{x^2 - x - 30} \geq 0$$

$$\Rightarrow \frac{x^2(x - 1)(x - 2)}{(x + 5)(x - 6)} \geq 0$$

$x \neq -5, 6$

$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$



Q.23 (2)

$\because$ $(m-2)x^2 + 8x + m + 4 > 0 \ \forall x \in R$

$\Rightarrow$ $m > 2$ & $D < 0$

$64 - 4(m-2)(m+4) < 0$

$16 - [m^2 + 2m - 8] < 0$

$\Rightarrow$ $m^2 + 2m - 24 > 0$

$\Rightarrow$ $(m+6)(m-4) > 0$

$m \in (-\infty, -6) \cup (4, \infty)$

But $m > 2$

$\Rightarrow$ $m \in (4, \infty)$

Then least integral m is m = 5.

Q.24 (2)

$-1 \le |x-1| - 1 \le 1$

$\Rightarrow$ $0 \le |x-1| \le 2$

$\Rightarrow$ $0 \le |x-1|$

$\Rightarrow$ $x \in R$ ...(1)

and $|x-1| \le 2$

$\Rightarrow$ $-2 \le x - 1 \le 2$

$\Rightarrow$ $-1 \le x \le 3$ ...(2)

(1) $\cap$ (2)

$\Rightarrow$ $x \in [-1, 3]$.

Q.25 (1)

$\log_{1/3} \dfrac{3x-1}{x+2} < 1$

$\Rightarrow$ $\dfrac{3x-1}{x+2} > 0 \Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{1}{3}, \infty\right)$ ....(i)

and $\dfrac{3x-1}{x+2} > \dfrac{1}{3}$

$\Rightarrow$ $\dfrac{8x-5}{x+2} > 0$

$\Rightarrow$ $x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right)$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right)$

Q.26 (2)

$2 - \log_2 (x^2 + 3x) \ge 0$

$\Rightarrow$ $\log_2 (x^2 + 3x) \le 2$

$x^2 + 3x > 0$

$\Rightarrow$ $x \in (-\infty, -3) \cup (0, \infty)$

.....(i)

and $x^2 + 3x \le 4$

$\Rightarrow$ $(x-1)(x+4) \le 0$

$\Rightarrow$ $x \in [-4, 1]$

.....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in [-4, -3) \cup (0, 1]$

Q.27 (4)

$\log_{1-x}(x-2) \ge 0$

$x > 2$ ....(1)

(i) When $0 < 1 - x < 1$ $\Rightarrow$ $0 < x < 1$

So no common range comes out.

(ii) When $1 - x > 1$ $\Rightarrow$ $x < 0$ but $x > 2$

here, also no common range comes out. , hence no solution.

Finally, no solution

Q.28 (1)

$\log_{0.3} (x-1) < \log_{0.09} (x-1)$

$\log_{0.3} (x-1) < \dfrac{\log_{0.3}(x-1)}{2}$

$\Rightarrow$ $\log_{0.3} (x-1) < 0 \Rightarrow x - 1 > 1$

$\Rightarrow$ $x > 2$

Q.29 (1)

$\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$

$\log_{0.5} \log_5 (x^2 - 4) > 0$

$\Rightarrow$ $x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

$\log_5 (x^2 - 4) > 0 \Rightarrow x^2 - 5 > 0$

$\Rightarrow$ $x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

$\log_5 (x^2 - 4) < 1$

$\Rightarrow$ $x^2 - 9 < 0 \Rightarrow x \in (-3, 3)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

Q.30 (4)

$\left(\dfrac{1}{2}\right)^{x^2-2x} < \left(\dfrac{1}{2}\right)^{2}$

here base is less than zero so inequality change

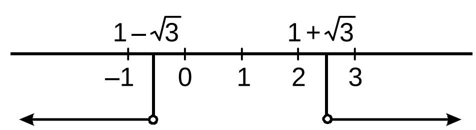
$\Rightarrow$ $x^2 - 2x > 2 \Rightarrow x^2 - 2x - 2 > 0$

$\alpha, \beta = \dfrac{2 \pm \sqrt{4+8}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2}$

$a = 1 - \sqrt{3}$, $b = 1 + \sqrt{3}$

$(x-a)(x-b) > 0$

$x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left(1+\sqrt{3}, \infty\right)$, x can be in $(3, \infty)$



Q.31 (1)

$\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$

D^r is always > 0

$6x^2 - 5x - 3 - 4x^2 + 8x - 24 \le 0$

$\Rightarrow$ $2x^2 + 3x - 27 \le 0$

$\Rightarrow$ $(2x+9)(x-3) \le 0 \Rightarrow x \in \left[\dfrac{-9}{2}, 3\right]$

least value of $4x^2 = 4.0^2 = 0$

Highest value of $4x^2$ is = max $\left(4\cdot\left(-\dfrac{9}{2}\right)^2, \ 4\cdot 3^2\right)$

= max (81, 36) = 81

Q.32 (3)

Let the roots be $\alpha, \beta, -\beta$

then $\alpha + \beta - \beta = p$

$\Rightarrow \alpha = p$ ...(1)

and $\alpha\beta - \alpha\beta - \beta^2 = q$

$\Rightarrow \beta^2 = -q$ ...(2)

also $-\alpha\beta^2 = r$

$\Rightarrow pq = r$ [using (1)].

Q.33 (3)

$x^3 - x - 1 = 0$ with roots α, β, γ

then $\alpha^3 - \alpha - 1 = 0$.
............(1)

Let $\frac{1+\alpha}{1-\alpha} = y \Rightarrow \alpha = \frac{y-1}{y+1}$

from equation (1) $\left(\frac{y-1}{y+1}\right)^3 - \left(\frac{y-1}{y+1}\right) - 1 = 0$

$\Rightarrow y^3 + 7y^2 - y + 1 = 0$ with roots $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$

then $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = -7$ Ans.

Q.34 (4)

$x^4 - 4x^3 + ax^2 + bx + 1 = 0$

real & positive roots

$\alpha + \beta + r + \delta = 4$ & $\alpha\beta r\delta = 1$

$\Rightarrow \alpha = \beta = r = \delta = 1$

$\Sigma\alpha\beta = a \Rightarrow a = 6$

$\Sigma\alpha\beta r = -b \Rightarrow b = -4$

or $(x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

Q.35 (4)

$ax^2 + bx + c = 0$ with roots α, β

sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5(\alpha + \beta) = 5\left(-\frac{b}{a}\right)$

Product of roots $= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$

$= 6\left(\frac{-b}{a}\right)^2 + \frac{c}{a} = \frac{6b^2}{a^2} + \frac{c}{a}$

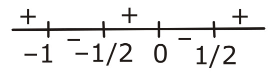
Q. E. $x^2 + \frac{5b}{a}x + \frac{6b^2}{a^2} + \frac{c}{a} = 0$

$a^2x^2 + 5abx + 6b^2 + ac = 0$

Q.36 (1)

$\Rightarrow \frac{(2x-1)}{x(2x^2+3x+1)} > 0$

$\Rightarrow \frac{(2x-1)}{x(x+1)(2x+1)} > 0$

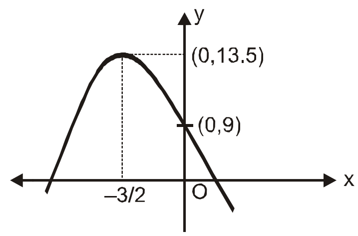


consontains $\left(-\infty, \frac{-3}{2}\right)$

Q.37 (3)

$y = -2x^2 - 6x + 9$

$\therefore \frac{-b}{2a} = \frac{6}{2(-2)} = -\frac{3}{2} = -1.5$



& $D = 36 - 4(-2)(9) = 36 + 72 = 108$

$\therefore -\frac{D}{4a} = -\frac{108}{4(-2)} = +\frac{108}{8} = 13.5$

$\Rightarrow y \in (-\infty, 13.5]$

Q.38 (1)

$k = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow (k-1)x^2 + (k+1)x + (k-1) = 0$

Q x is real

$\therefore D \geq 0$

$\Rightarrow (k+1)^2 - 4(k-1)^2 \geq 0$

$\Rightarrow (3k-1)(k-3) \leq 0$

$\Rightarrow k \in \left[\frac{1}{3}, 3\right]$

Q.39 (3)

$y = \frac{2x}{1+x^2}, x \in R$

$\Rightarrow yx^2 - 2x + y = 0$

$\Rightarrow D \geq 0 \Rightarrow 4 - 4y^2 \geq 0$

$\Rightarrow (y^2 - 1) \leq 0 \Rightarrow y \in [-1, 1]$

$\therefore$ Range of $f(y) = y^2 + y - 2$

Min value $= \frac{-D}{4a} = \frac{-9}{4}$ at $y = \frac{-b}{2a} = \frac{-1}{2}$

$y = \frac{-1}{2} \in [-1, 1]$

$f(-1) = 1 - 1 - 2 = -2$

$f(1) = 1 + 1 - 2 = 0$

max value is $= 0$

Range $\left[\frac{-9}{4},\ 0\right]$

Q.40 (2)
$x^2 - xy + y^2 - 4x - 4y + 16 = 0$, $x, y \in R$
$x^2 - x(y + 4) + (y^2 - 4y + 16) = 0$
… (1)
$x \in R \Rightarrow D \geqslant 0$
$(y + 4)^2 - 4(y^2 - 4y + 16) \geqslant 0$
$\Rightarrow y^2 + 8y + 16 - 4y^2 + 16y - 64 \geqslant 0$
$\Rightarrow y^2 - 8y + 16 \leqslant 0 \quad \Rightarrow (y - 4)^2 \leqslant 0 \Rightarrow y = 4$
Put is given equation (i)
$x^2 - 8x + 16 = 0$
$\Rightarrow (x - 4)^2 = 0 \Rightarrow x = 4$

Q.41 (4)
$(y - 1)x^2 + (y + 1)x + (2cy - c) = 0$
$D \geqslant 0 \therefore x \in R$
$\Rightarrow (y + 1)^2 - 4(y - 1)(2cy - c) \geqslant 0$
$y^2 + 2y + 1 - 8cy^2 + 12cy - 4c \geqslant 0$
$(1 - 8c)y^2 + (2 + 12c)y + (1 - 4c) \geqslant 0$
$\forall y \in R, D \leqslant 0$
$(2 + 12c)^2 - 4(1 - 8c)(1 - 4c) \leqslant 0$
$(1 + 6c)^2 - (1 - 8c)(1 - 4c) \leqslant 0$
$4c^2 + 24c \leqslant 0 \Rightarrow c \in [-6, 0]$
& N^r & D^r have no any common root
(i) both common factor (root) (not possible)

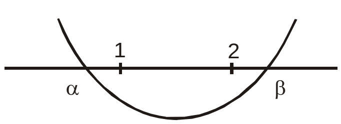
$$\frac{1}{1} = \frac{-1}{+1} = \frac{c}{2c}$$

(ii) If one common root is α
$(\alpha^2 - \alpha + c = 0) \times 2$
& $\alpha^2 + \alpha + 2c = 0$

$\alpha^2 - 3\alpha = 0$
$\alpha = 0 \Rightarrow c = 0$
or $\alpha = 3 \Rightarrow c = -6$
$\therefore$ $c \neq 0$ & $c \neq -6$
$\therefore$ $c \in (-6, 0)$

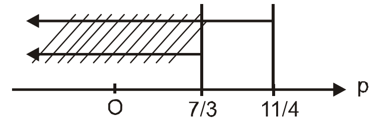
Q.42 (2)
$2x^2 - (a^3 + 8a - 1)x + (a^2 - 4a) = 0$
since the roots are of opposite sign,
$f(0) < 0$
$\Rightarrow a^2 - 4a < 0$
$\Rightarrow a(a - 4) < 0$
$\Rightarrow a \in (0, 4)$

Q.43 (2)
$x^2 - 2px + (8p - 15) = 0$
$f(1) < 0$ and $f(2) < 0$
$\Rightarrow f(1) = 1 - 2p + 8p - 15 < 0$



$\Rightarrow p < 7/3$
and $f(2) = 4 - 4p + 8p - 15 < 0$

$\Rightarrow 4p - 11 < 0 \Rightarrow p < \frac{11}{4}$

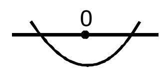


Hence $p \in (-\infty, 7/3)$ Ans.

Q.44 *(4)*
$x^2 + 2(k - 1)x + k + 5 = 0$
Case - I
(i) D … 0
$\Rightarrow$ $4(k - 1)^2 - 4(k + 5)$ … 0
$\Rightarrow$ $k^2 - 3k - 4$ … 0 $\Rightarrow (k + 1)(k - 4)$ … 0
$\Rightarrow$ $k \in (-\infty, -1] \cup [4, \infty)$
& (ii) $f(0) > 0 \Rightarrow k + 5 > 0 \Rightarrow k \in (-5, \infty)$

& (iii) $\frac{-b}{2a} > 0 \Rightarrow \frac{-2(k-1)}{2} > 0$
$\Rightarrow k \in (-\infty, 1) \therefore k \in [-5, -1]$

Case - II $f(0) \leq 0 \Rightarrow k + 5 \leq 0$
$\Rightarrow k \in (-\infty, -5]$



Finally $k \in$ (Case - I) $\cup$ (Case - II)
$k \in (-\infty, -1]$

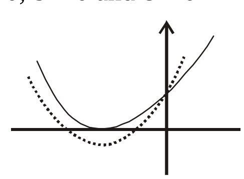
JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 *(A)*
$\pi^x = -2x^2 + 6x - 9$
$D = 36 - 4(-2)(-9) = 36 - 72 < 0$ & $a < 0$
So quadratic expression $-2x^2 + 6x - 9$ is always negative whereas π^x is always +ve
$\therefore$ Equation will not hold for any x. $\therefore$ $x \in \phi$
So $\pi^x = -2x^2 + 6x - 9$ has no solution.

Q.2 (C)
Given equation
$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$
can be re-written as
$3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$
$\Delta = 4\{(a + b + c\}^2 - 3(ab + bc + ca)\}$ ($\because$ $b^2 - 4ac = \Delta$)
$= 4(a^2 + b^2 + c^2 - ab - bc - ac)$
$= 2\{(a - b)^2 + (b - c)^2 + (c - a)^2\} \geq 0$
Hence both roots are always real.

Q.3 (B)
$a > 0$, $b > 0$ and $c > 0$



$ax^2 + bx + c = 0$ ⟨ α, β

$\alpha + \beta = -b/a = -$ ve,

$\alpha\beta = \frac{c}{a}$ = + ve –ve real part

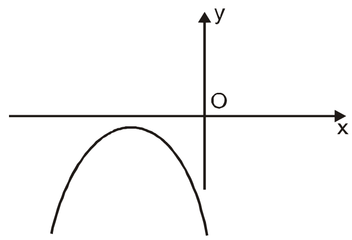
Q.4 (B)

$\because$ roots of $ax^2 + bx + c = 0$ are not real $\Rightarrow D < 0$

Also given, $a - b + c < 0$

$\Rightarrow f(-1) < 0$

$\Rightarrow$ Possible graph is as shown



$\therefore f(x) < 0 \ \forall \ x$

$\Rightarrow f(-2) < 0$

$\Rightarrow 4a + c < 2b.$

Q.5 (B)

$(\lambda + 2)(\lambda - 1)x^2 + (\lambda + 2)x - 1 < 0 \ \forall \ x \in R$

$\Rightarrow (\lambda + 2)(\lambda - 1) < 0$

$\Rightarrow -2 < \lambda < 1$ ...(1)

$(a < 0)$ and $(\lambda + 2)^2 + 4(\lambda + 2)(\lambda - 1) < 0 \ (D < 0)$

$\Rightarrow (\lambda + 2)(\lambda + 2 + 4\lambda - 4) < 0$

$\Rightarrow (\lambda + 2)(5\lambda - 2) < 0$

$\Rightarrow -2 < \lambda < \frac{2}{5}$ ...(2)

(1) & (2) $\Rightarrow \lambda \in \left(-2, \frac{2}{5}\right)$

Also $\lambda = -2 \Rightarrow 0 < 1$ which is true

$\therefore$ Required interval is

$\lambda \in \left[-2, \frac{2}{5}\right).$

Q.6 (A)

$x^2 + px + 12 = 0$

$\Rightarrow 4^2 + p4 + 12 = 0 \Rightarrow 4p = -28 \Rightarrow p = -7$

Now second equation

$\Rightarrow x^2 - 7x + q = 0$ has equal roots

$\Rightarrow D = 0 \Rightarrow 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$

Q.7 (B)

$a \leqslant 0 \ x^2 - 2a|x - a| - 3a^2 = 0$

If $x = a \Rightarrow a^2 - 3a^2 = 0 \Rightarrow a = 0 \ x > a$

$\Rightarrow x^2 - 2ax - a^2 = 0$

$$x = \frac{2a \pm \sqrt{4a^2 + 4a^2}}{2} = a \pm \sqrt{2}a$$

$a + \sqrt{2}a < a \Rightarrow x \neq a + \sqrt{2}a$

or $a - \sqrt{2}a > a \therefore x = (1 - \sqrt{2})a$

$x < a$

$x^2 + 2ax - 5a^2 = 0$

$$x = \frac{-2a \pm \sqrt{4a^2 + 20a^2}}{2} = \frac{-2a \pm 2\sqrt{6}a}{2}$$

$x = -a \pm \sqrt{6}\,a$

$x \neq -a(1 + \sqrt{6}) \quad (\because x < 0)$

or $-a + \sqrt{6}\,a < a \ \therefore \ x = (-1 + \sqrt{6})a$

Q.8 (B)

$mx^2 - 9mx + 5m + 1 > 0 \ \forall \ x \in R$

$D < 0$ & $m > 0$

$81m^2 - 4m(5m + 1) < 0$

$81m^2 - 20m^2 - 4m < 0$

$61m^2 - 4m < 0$

$m(61m - 4) < 0 \Rightarrow m \in \left(0, \frac{4}{61}\right)$

If $m = 0 \Rightarrow 1 > 0 \ \forall \ x \in R \Rightarrow m = 0$

$m \in \left[0, \frac{4}{61}\right)$

Q.9 (D)

Sum of roots < 1

$\Rightarrow \lambda^2 - 5\lambda + 5 < 1$

$\Rightarrow (\lambda - 1)(\lambda - 4) < 0$

$\Rightarrow 1 < \lambda < 4$ ...(1)

$\Rightarrow$ Product of roots < 1

$\Rightarrow 2\lambda^2 - 3\lambda - 5 < 0$

$\Rightarrow (2\lambda - 5)(\lambda + 1) < 0$

$\Rightarrow -1 < \lambda < \frac{5}{2}$ ...(2)

(1) & (2)

$\Rightarrow 1 < \lambda < \frac{5}{2}$

Q.10 (C)

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow (\alpha + \beta) = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \Rightarrow \frac{-b}{a} = \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2}{c^2} - \frac{2a}{c} \Rightarrow \frac{2a}{c} = \frac{b^2}{c^2} + \frac{b}{a}$$

$\Rightarrow \frac{2a}{c} = \frac{ab^2 + bc^2}{ac^2}$, $2a^2c = ab^2 + bc^2$ then dividing by abc

$$\Rightarrow \frac{2a^2c}{abc} = \frac{ab^2}{abc} + \frac{bc^2}{abc}$$

$$\Rightarrow \frac{2a}{b} = \frac{b}{c} + \frac{c}{a} \Rightarrow \frac{2}{\left(\frac{b}{a}\right)} = \frac{1}{\left(\frac{c}{b}\right)} + \frac{1}{\left(\frac{a}{c}\right)}$$

$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ in H.P.

Q.11 (A)

By given condition

$\alpha + 2\alpha = \frac{-(3a-1)}{(a^2-5a+3)}$ & $2\alpha^2 = \frac{2}{(a^2-5a+3)}$

$\Rightarrow \alpha^2 = \frac{(3a-1)^2}{9(a^2-5a+3)^2} = \frac{1}{(a^2-5a+3)}$

$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$

$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$

Q.12 (B)

Let $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a} = \alpha^2 + \beta^2$

$\Rightarrow (\alpha + \beta)[(\alpha + \beta) - 1] = 2\alpha\beta$

$\alpha\beta = \frac{c}{a} = \alpha^2\beta^2$

$\alpha\beta(\alpha\beta - 1) = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha\beta = 1$

If $\alpha\beta = 0$ $(\alpha + \beta)[(\alpha + \beta) - 1] = 0$

$\alpha + \beta = 0, \alpha + \beta = 1$

If $\alpha\beta = 1$ $(\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0$

$((\alpha + \beta) - 2)((\alpha + \beta) + 1) = 0$

$\alpha + \beta = 2, \alpha + \beta = -1$

Quadratic Equations are

(1) $x^2 + 0x + 0 = 0$

(2) $x^2 - x + 0 = 0$

(3) $x^2 - 2x + 1 = 0$

(4) $x^2 + x + 1 = 0$

Q.13 (A)

$x^3 + 5x^2 + px + q = 0$ (roots α, β, x_1) $\Rightarrow \alpha + \beta + x_1 = -5$, $\alpha\beta + \beta x_1 + \alpha x_1 = p$ ...(1)

$x^3 + 7x^2 + px + r = 0$ (roots α, β, x_2) $\Rightarrow \alpha + \beta + x_2 = -7$, $\alpha\beta + \beta x_2 + \alpha x_2 = p$ ...(2)

Subtracting (2) from (1)

$\alpha\beta + \beta x_1 + \alpha x_1 = p$

$\alpha\beta + \beta x_2 + \alpha x_2 = p$

$\alpha(x_1 - x_2) + \beta(x_1 - x_2) = 0$

$(x_1 - x_2)(\alpha - \beta) = 0$ $[x_1 \neq x_2]$

$\therefore$ $\alpha + \beta = 0 \Rightarrow x_1 = -5$

$x_2 = -7$

Q.14 (B)

Let common roots is α

Using cross multiplication rule

$\frac{\alpha^2}{a-b} = \frac{\alpha}{2-3} = \frac{1}{3b-2a}$

$\alpha = \frac{a-b}{-1}$ & $\alpha = \frac{-1}{3b-2a}$

$\alpha = b - a = \frac{1}{2a-3b}$

$\Rightarrow (b - a)(2a - 3b) = 1$

$\Rightarrow 5ab - 2a^2 - 3b^2 = 1$ Ans.

Q.15 (A)

$$\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right)} \geq 0$$

For $\sqrt{(x-8)(2-x)}$ to be defined

(i) $(x - 8)(2 - x) \geq 0$

$(x - 2)(x - 8) \leq 0$ $\Rightarrow 2 \leq x \leq 8$

Now Let say $y = \log_{0.3} \frac{10}{7}(\log_2 5 - \log_2 2) = \log_{0.3} \frac{10}{7}(\log_2 5/2)$

Let $y < 0$ (assume)

then $\log_{0.3} \frac{10}{7}(\log_2 5/2) < 0$

$\Rightarrow \frac{10}{7}\log_2 5/2 > 1 \Rightarrow \log_2 5/2 > \frac{7}{10}$

$\Rightarrow \frac{5}{2} > 2^{(7/10)}$ which is true

So $y < 0$

so denominator is – ve and numerator is +ve, so inequality is not satisfied,

thus $\sqrt{(x-8)(2-x)} = 0$

$x = 2, 8$ .....(i)

Now $2^{x-3} > 31$

$\Rightarrow (x - 3) > \log_2 31 \Rightarrow x > 3 + \log_2 2^{4.9}$ (approx)

$\Rightarrow x > 7.9$

$\Rightarrow x = 8$

Q.16 (B)

$(x - a)(x - b)(x - c) = d$ (roots α, β, γ); $d \neq 0$

Now

$(x - \alpha)(x - \beta)(x - \gamma) + d = 0$

$\Rightarrow (x - a)(x - b)(x - c) - d + d = 0$

$\Rightarrow (x - a)(x - b)(x - c) = 0$ roots are a, b, c

Q.17 (B)

$x^4 - Kx^3 + Kx^2 + Lx + M = 0$ (roots $\alpha, \beta, \gamma, \delta$)

$\sum \alpha = K$, $\sum \alpha\beta = K$, $\sum \alpha\beta\gamma = -L$

$\alpha\beta\gamma\delta = M$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2\sum \alpha\beta$

$K^2 - 2K = (K - 1)^2 - 1$

$(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)_{min} = -1$

Q.18 *(A)*

Let α, β, γ be the roots of $x^3 - Ax^2 + Bx - C = 0$ ...(1)

the roots of $x^3 + Px^2 + Qx - 19 = 0$ will be $(\alpha + 1), (\beta + 1), (\gamma + 1)$

$\therefore \ (\alpha + 1)(\beta + 1)(\gamma + 1) = 19$

$\Rightarrow \ (\alpha\beta + \alpha + \beta + 1)(\gamma + 1) = 19$

$\Rightarrow \ \alpha\beta\gamma + \alpha\gamma + \beta\gamma + \alpha\beta + \alpha + \beta + \gamma + 1 = 19$

$\Rightarrow$ C + B + A = 18 [using (1)].

Q.19 (B)

$a^2x^4 + bx^3 + cx^2 + dx + f^2$ is perfect square

in the form

$= (ax^2 + mx + f)^2$

$= a^2x^4 + (2am)x^3 + (m^2 + 2af)x^2 + 2mfx + f^2$

by comparision 2am = b,

$m = \dfrac{b}{2a}$ substituting the value of m in equation (2) , c

$= m^2 + 2af$, $c = \dfrac{b^2}{4a^2} + 2af$ $4a^2c = b^2 + 8a^3f$

$4a^2c - b^2 = 8a^3f$

Q.20 (C)

$a^2 + b^2 + c^2 = 1$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\Sigma ab$

$= 1 + 2\Sigma ab$

$$\Sigma ab = \frac{(a+b+c)^2 - 1}{2}$$

$$\Rightarrow \text{Min } \Sigma ab = \frac{0-1}{2} = -\frac{1}{2}$$

Now $\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] \geqslant 0$

$a^2 + b^2 + c^2 - ab - bc - ca \geqslant 0 \ \Rightarrow \ 1 \geqslant \Sigma ab$

$$\therefore \ \Sigma ab \in \left[-\frac{1}{2}, 1\right]$$

Q.21 (B)

$(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$

$(a - 1)(x^2 + x + 1)^2$

$- (a + 1)(x^2 + x + 1)(x^2 - x + 1) = 0$

$(x^2 + x + 1)[(a - 1)(x^2 + x + 1)$

$- (a + 1)(x^2 - x + 1)] = 0$

$x^2 + x + 1 > 0 \ \forall \ x \in R$

$\Rightarrow \ (a - 1)(x^2 + x + 1) - (a + 1)(x^2 - x + 1) = 0$

$\Rightarrow \ -2x^2 + 2ax - 2 = 0 \quad \Rightarrow \ x^2 - ax + 1 = 0$

$D > 0 \ \Rightarrow \ a^2 - 4 > 0$

$a \in (-\infty, -2) \cup (2, \infty)$

Q.22 (B)

Case - I $b > 0 \Rightarrow ax^2 + 2bx + b > 0$

$a > 0, D < 0$

$4b^2 - 4ab < 0$

$(b^2 - ab) < 0$

Case - II $b < 0 \Rightarrow ax^2 + 2bx + b < 0$

$a < 0, \ D < 0$

$4b^2 - 4ab < 0$

$b^2 - ab < 0$

In both case $(b^2 - ab) < 0$

Now $bx^2 + (b - c)x + b - c - a = 0$

$D = (b - c)^2 - 4b(b - c - a)$

$D = b^2 + c^2 - 2bc - 4b^2 + 4bc + 4ab$

$D = (b + c)^2 - 4(b^2 - ab)$

$(b + c)^2 > 0$ & $(b^2 - ab) < 0$

$\Rightarrow$ D > 0 $\Rightarrow$ real & Distinct root

Q.23 (D)

$x^2 - (a - 2)x - a - 1 = 0$ with roots α, β

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6$

Min $(\alpha^2 + \beta^2)$ at $\dfrac{-B}{2A} = \dfrac{+2}{2} = 1 \quad \Rightarrow a = 1$

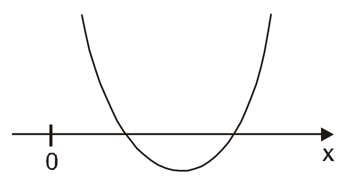
Q.24 (B)

$(2 - x)(x + 1) = p$

$\Rightarrow x^2 - x + (p - 2) = 0$ ...(1)

(1) has both roots distinct & positive

$\therefore$ (i) D > 0 (ii) f(0) > 0 (iii) $\dfrac{-b}{2a} > 0$



(i) $D > 0 \Rightarrow p < \dfrac{9}{4}$

(ii) $f(0) > 0 \Rightarrow p > 2$

(iii) $\dfrac{-b}{2a} = \dfrac{1}{2} > 0$ (always true)

$\therefore$ (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow p \in \left(2, \dfrac{9}{4}\right)$.

Q.25 (D)

$4x^2 - 16x + \lambda = 0, \lambda \in R$

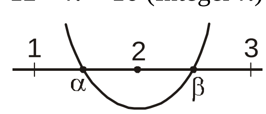
$1 < \alpha < 2$ & $2 < \beta < 3$

f(1) f(2) < 0 & f(2) f(3) < 0

$(-12 + \lambda)(-16 + \lambda)$

& $(-16 + \lambda)(-12 + \lambda) = 0$

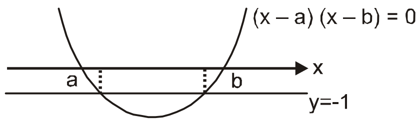
$12 < \lambda < 16$ (Integer λ) = 13, 14, 15 Three values

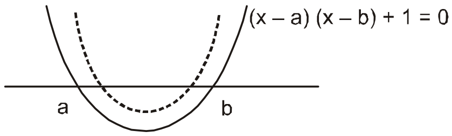


Q.26 (A)

b > a

both roots lies in (a, b)



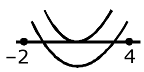


Q.27 (B)

$x^2 - 2mx + m^2 - 1 = 0$

(i) $D \geqslant 0$

$4m^2 - 4(m^2 - 1) \geqslant 0$



$\Rightarrow\ 4 \geqslant 0 \Rightarrow m \in R$ ...(1)

& (ii) $f(-2) > 0$

$4 + 4m + m^2 - 1 > 0 \Rightarrow m^2 + 4m + 3 > 0$

$(m + 3)(m + 1) = 0 \Rightarrow m \in (-\infty, -3) \cup (-1, \infty)$ ....(2)

& (iii) $f(4) > 0$

$16 - 8m + m^2 - 1 > 0$

$m^2 - 8m + 15 > 0$

$(m - 3)(m - 5) > 0$

$\Rightarrow m \in (-\infty, 3) \cup (5, \infty)$ .....(3)

& (iv) $-2 < \frac{-b}{2a} < 4$

$-2 < \frac{2m}{2} < 4 \Rightarrow m \in (-2, 4)$ ....(4)

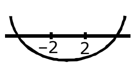
taking intersection of (1), (2), (3) & (4)

finally $m \in (-1, 3)$

Q.28 (D)

$ax^2 + bx + c = 0 \quad a, b, c \in R$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$



$\Rightarrow f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$

(1) $f(-2) < 0$ & (2) $f(+2) < 0$

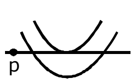
$4a - 2b + c < 0 \quad 4a + 2b + c < 0$

$4 - \frac{2b}{a} + \frac{c}{a} < 0 \quad 4 + \frac{2b}{a} + \frac{c}{a} < 0$

Q.29 (B)

(i) $D \geqslant 0$

$1 - 4p \geqslant 0 \Rightarrow p \leq \frac{1}{4}$ ....(1)



& (ii) $f(p) > 0$

$p^2 + p + p > 0 \Rightarrow p(p + 2) = 0$

$\Rightarrow\ p \in (-\infty, -2), \cup (0, \infty)$ ....(2)

& (iii) $\frac{-b}{2a} > p \Rightarrow -\frac{1}{2} > p$ ....(3)

taking intersection of (1), (2) & (3)

finally $p \in (-\infty, -2)$

Q.30 (C)

$3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0 \quad f(0) < 0$

$\Rightarrow k^2 - 3k + 2 < 0$

$\Rightarrow\ (k - 2)(k - 1) < 0 \Rightarrow\ k \in (1, 2)$

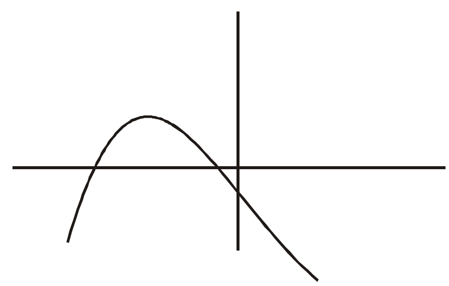
JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 *(A, B, D)*

$y = ax^2 + bx + c$

Clearly $a < 0$

and $\frac{-b}{2a} < 0$



$\Rightarrow b < 0$

also $f(0) < 0 \Rightarrow c < 0$

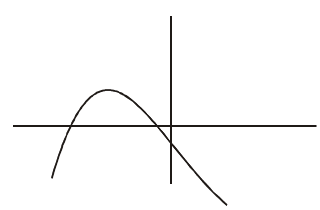
and $D > 0$

$\therefore$ (A), (B) and (D).

Q.2 (A,B,C,D)

(A) $a < 0$,

$-\frac{-b}{2a} < 0 \Rightarrow b < 0$

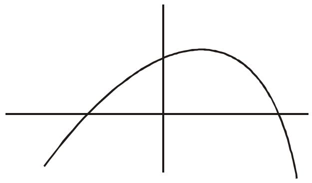


& $f(0) < 0 \Rightarrow c < 0$

$\therefore$ $abc < 0$

(B) $a < 0$,

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

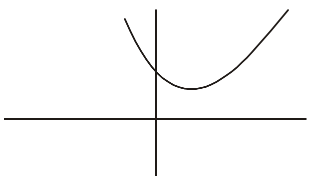


f(0) > 0 ⇒ c > 0

⇒ abc < 0

(C) a > 0

$\frac{-b}{2a} > 0 \Rightarrow b < 0$

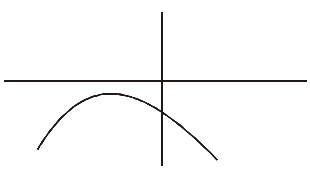


f(0) > 0 c > 0

⇒ abc < 0

(D) a < 0

$\frac{-b}{2a} < 0 \Rightarrow b < 0$



f(0) < 0 ⇒ c < 0

∴ (A), (B), (C), (D)

Q.3 (A,D)

Clearly a < 0

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

∴ (A), (D)

Q.4 (A, B, D)

$|x|^2 + |x| - 6 = 0 \Rightarrow |x| = -3, 2 \Rightarrow |x| = 2$

$\Rightarrow x = \pm 2$

Q.5 (B,D)

$ax^2 + bx + c = 0 \quad \alpha, \beta$

a + b = –b/a , ab = c/a

$px^2 + qx + r = 0 \quad \alpha + h, \beta + h$

$(a + b) + 2h = \frac{-q}{p}$

$$h = \frac{\frac{-q}{p} + \frac{b}{a}}{2} = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right) \text{ Ans.}$$

$$|\alpha - \beta| = |(\alpha + h) - (\beta + h)|$$

$$= \sqrt{[(\alpha + h) + (\beta + h)]^2 - 4(\alpha + h)(\beta + h)}$$

$$= \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p} \Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$$

Q.6 (B,C,D)

(A) $S = a^2 + b^2 = a^2 - 2b$

$P = a^2 b^2 = b^2$

∴ equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

(B) $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{b}$

$\therefore x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow bx^2 + ax + 1 = 0$

(C) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{a^2 - 2b}{b}$

$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2 - 2b}{b}x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b)x + b = 0$

(D) $S = a + b - 2 = -a - 2$

$P = (a - 1)(b - 1)$

$= ab - (a + b) + 1$

$= b + a + 1$

∴ equation is

$x^2 + (a + 2)x + (a + b + 1) = 0.$

Q.7 (A,D)

$ax^3 + bx^2 + cx + d = 0 \quad \alpha, \beta, \gamma$

Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$

Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β

So the third root 'γ' will be real.

$\alpha + \beta + \gamma = \frac{-b}{a}$

$-1 + \gamma = \frac{-b}{a}$

$\gamma = \frac{a - b}{a}$

Also $\alpha\beta\gamma = \frac{-d}{a}$

But $\alpha\beta = 1$

$\therefore\ \gamma = \frac{-d}{a}$

$\therefore$ Ans are (A) & (D).

Q.8 (A,B,D)

$\frac{1}{2} \le \log_{1/10} x \le 2$

$\Rightarrow\ \frac{1}{100} \le x \le \frac{1}{\sqrt{10}}$

Q.9 (B, D)

$x^2 + abx + c = 0 \begin{cases}\alpha \\ \beta\end{cases}$ ...(1)

$\alpha + \beta = -ab,\ \alpha\beta = c$

$x^2 + acx + b = 0 \begin{cases}\alpha \\ \delta\end{cases}$ ...(2)

$\alpha + \delta = -ac,\ \alpha\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c-b} = \frac{1}{ac - ab}$

$\Rightarrow \alpha^2 = \frac{a(b^2 - c^2)}{a(c-b)} = -(b+c)$

$\&\ \alpha = \frac{c-b}{a(c-b)} = \frac{1}{a}$ $\therefore$ common root, $\alpha = \frac{1}{a}$

$\therefore\ -(b+c) = \frac{1}{a^2} \Rightarrow$ $a^2(b+c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \frac{1}{a} = c \Rightarrow \beta = ac$

$\&\ \delta \times \frac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is

$x^2 - a(b+c)x + a^2bc = 0$

$a(b+c)x^2 - a^2(b+c)^2x + a.(b+c)a^2bc = 0$

$a(b+c)x^2 + (b+c)x - abc = 0.$

Q.10 (C,D)

$\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary $\Rightarrow$ both the roots of quadratic are same

$\Rightarrow\ b^2 - 4ac < 0\ \&\ \frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$

$\Rightarrow$ a = k, b = 4k, c = 5k.

Q.11 (A, D)

$x^2 + px + q = 0 \begin{cases}\alpha \\ \beta\end{cases}$

$\alpha + \beta = -p,\ \alpha\beta = q$ and $p^2 - 4q > 0$

$x^2 - rx + s = 0 \begin{cases}\alpha^4 \\ \beta^4\end{cases}$

.....(1)

Now $\alpha^4 + \beta^4 = r$

$\Rightarrow\ \alpha^4 + \beta^4 = r,\ (\alpha\beta)^4 = s = q^4$

$\therefore\ (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r \Rightarrow [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r$

$\Rightarrow (p^2 - 2q)^2 - 2q^2 = r$

$\Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$

.....(2)

Now, for $x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$ by equation (2)

$= 8q^2 + 4r = 4(2q^2 + r) > 0$

$\Rightarrow$ D > 0 two real and distinct roots

Product of roots $= 2q^2 - r$

$= 2q^2 - [(p^2 - 2q)^2 - 2q^2]$

$= 4q^2 - (p^2 - 2q)^2$

$= -p^2(p^2 - 4q) < 0$ from (1)

So product of roots is – ve

hence roots are opposite in sign

Q.12 (A,D)

$20x^2 + 210x + 400 = 4500 \Rightarrow 2x^2 + 21x - 410 = 0$

$\Rightarrow\ (2x + 41)(x - 10) = 0$

$\Rightarrow\ x = \frac{-41}{2},\ x = 10 \quad \Rightarrow\ x = -20.5,\ x = 10$

Q.13 (A,B,C,D)

$x^3 + bx^2 + cx - 1 = 0 \begin{cases}\alpha = \frac{a}{r} \\ \beta = a \\ r = ar\end{cases}$

$\frac{a}{r} + a + ar = -b \ \Rightarrow\ a\left(\frac{1}{r} + 1 + r\right) = -b$

$\&\ \frac{a}{r} \times a \times ar = 1$

$a^3 = 1 \Rightarrow\ a = 1$

$\&\ \frac{a}{r}a + a\cdot ar + \frac{a}{r}\cdot ar = c$

$a^2\left(\frac{1}{r} + r + 1\right) = c$

$\frac{1}{r} + r + 1 = -b\ \&\ \frac{1}{r} + r + 1 = c \Rightarrow b + c = 0$

we know $\frac{1}{r} + r > 2 \Rightarrow \left(\frac{1}{r} + r + 1\right) > 3$

$-b > 3 \Rightarrow b < -3 \quad \Rightarrow b \in (-\infty, -3)$

& other two roots are $\frac{1}{r}$ & r

if $\frac{1}{r} > 1 \Rightarrow r < 1$ if $r > 1 \Rightarrow r < 1$

Q.14 (A,B)

$$f(x) = \frac{3}{(x-2)} + \frac{4}{(x-3)} + \frac{5}{(x-4)} = 0$$

$6x^2 - 14x - 21x + 49 = 0$

$(3x - 7)(2x - 7) = 0$

$x = \frac{7}{2}, x = \frac{7}{2}$

$2 < \frac{7}{2} < 33 < \frac{7}{2} < 4$

2nd Method

$g(x) = 3(x - 3)(x - 4) + 4(x - 2)(x - 4) + 5(x - 2)(x - 3) = 0$

$g(2) > 0; g(3) < 0, g(4) > 0$

one root lie b/w (2, 3)

& other root lie b/w (3, 4)

Comprehension # 1 (Q. No. 15 & 16)

15. (D)

$f(-1) = 0 \quad \Rightarrow -1 + q - r = 0$

$f(-5) = 0 \quad \Rightarrow -25 + 5q - r = 0$

$q = 6$

$r = 5$

$f(x) = -x^2 - 6x + 5$ vertex is $(-3, 4)$

16. (B)

$f(x) = px^2 - qx - r$ Since $f(0)\,f(1) > 0 \Rightarrow (-r)(p - q - r) > 0 \Rightarrow r(p - q - r) < 0$

Comprehension # 2 (Q. No. 17 & 18)

17. (A)

18. (A)

(17 & 18)

Let the coordinates of $A(\alpha, 0)$, $B(2\alpha, 0)$, $C(0, 2\alpha)$.

Now $y = x^2 + bx + c$ passes through $C(0, 2\alpha)$

$\therefore$ given equation of curve reduces to $y = x^2 + bx + 2\alpha$. Now it also passes through A & B

$\therefore \quad 0 = \alpha^2 + b\alpha + 2\alpha \quad \Rightarrow$

$0 = \alpha + b + 2 \quad$ ..... (i)

$\& \quad 0 = 4\alpha^2 + 2\alpha b + 2\alpha \quad \Rightarrow$

$0 = 2\alpha + b + 1 \quad$ ..... (ii)

On solving (i) & (ii) for α & b we get $\alpha = 1, b = -3$

$\therefore$ given curve is $y = x^2 - 3x + 2$

17. roots of y = 0 are {2, 1}

18. $(\alpha + \beta) \Rightarrow 3 \quad (\because \alpha = 2, \beta = 1)$

$\Rightarrow (\alpha - \beta) \Rightarrow 1$

$\therefore$ equation whose roots are 3, 1 is $x^2 - 4x + 3 = 0$

Comprehension # 3 (Q. No. 19 to 21)

19. (C)

20. (B)

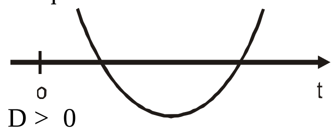
21. (D)

(19 to 21)

$x^4 - \lambda x^2 + 9 = 0 \quad \Rightarrow \quad x^2 = t \geq 0$

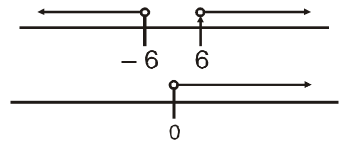
$\Rightarrow f(t) = t^2 - \lambda t + 9 = 0$

19. given equation has four real & distinct roots



$D > 0$

$\Rightarrow \lambda^2 - 36 > 0$



$\frac{-b}{2a} > 0$

$\Rightarrow \frac{\lambda}{2} > 0$

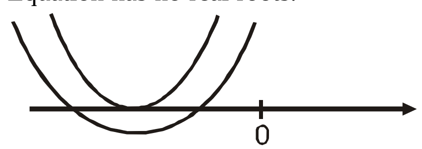
$\Rightarrow \lambda > 0$

$f(0) > 0$

$\Rightarrow 9 > 0$

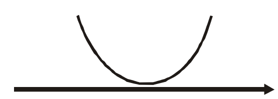
$\therefore \lambda \in (6, \infty)$

20. Equation has no real roots.



case-I $\quad D \geq 0 \Rightarrow \lambda^2 - 36 \geq 0$

$\frac{-b}{2a} < 0 \quad \Rightarrow \lambda < 0$



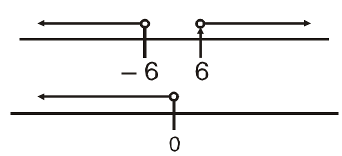
$f(0) > 0 \quad \Rightarrow 9 > 0.$

$\therefore \quad \lambda \in (-\infty, -6]$

case-II $\quad D < 0$

$\Rightarrow \lambda^2 - 36 < 0$

$\Rightarrow \lambda \in (-6, 6)$



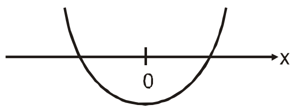
union of both cases gives

$\lambda \in (-\infty, 6)$

21. Equation has only two real roots

case-I $f(0) < 0$

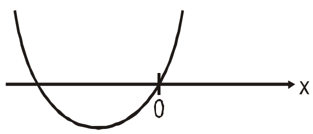
$9 < 0$



which is false

case-II $f(0) = 0$

and $\frac{-b}{2a} < 0$



∴ No solution

∴ Final answer is ϕ

Comprehension # 4 (Q.22 to 24)

22. (B); **23.** (A); **24.** (C)

Sol. 22 Divide by $x^2 \Rightarrow x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0 \Rightarrow x^2$

$+ \frac{1}{x^2} - 10\left(x + \frac{1}{x}\right) + 26 = 0$

$t = x + \frac{1}{x} \Rightarrow \quad t^2 - 2 = x^2 + \frac{1}{x^2}$

$\Rightarrow t^2 - 2 - 10t + 26 = 0$

$\Rightarrow t^2 - 10t + 24 = 0 < \begin{matrix} 4 \\ 6 \end{matrix}$

$t = 4$

$x + \frac{1}{x} = 4 \quad \Rightarrow x^2 - 4x + 1 = 0$

$\Rightarrow x = 2 \pm \sqrt{3}$

$t = 6 \quad x + \frac{1}{x} = 6 \quad \Rightarrow x^2 - 6x + 1 = 0$

$\Rightarrow x = 3 \pm 2\sqrt{2}$.

Sol.23 By trail x = 1 is a root divide by x – 1

1	1	–5	9	–9	5	–1
	×	1	–4	5	–4	1
	1	–4	5	–4	1	0

$(x - 1)(x^4 - 4x^3 + 5x^2 - 4x + 1) = 0$

$\Rightarrow x = 1$

or

$x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

$x^2 - 4x + 5 - \frac{4}{x} + \frac{1}{x^2} = 0$

$\Rightarrow t = x + \frac{1}{x}$

$\Rightarrow t^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow t^2 - 2 - 4t + 5 = 0$

$\Rightarrow t^2 - 4t + 3 = 0 < \begin{matrix} 1 \\ 3 \end{matrix}$

$\Rightarrow x + \frac{1}{x} = 1, x + \frac{1}{x} = 3$

$x^2 - x + 1 = 0, \quad x^2 - 3x + 1 = 0$

$\Rightarrow x = \frac{1 \pm i\sqrt{3}}{2}, x = \frac{3 \pm \sqrt{5}}{2}$

∴ roots are $1, \frac{1 \pm i\sqrt{3}}{2}, \frac{3 \pm \sqrt{5}}{2}$.

Sol.24 Divide by x^3

$\Rightarrow x^3 - 4x + \frac{4}{x} - \frac{1}{x^3} = 0$

$\Rightarrow x^3 - \frac{1}{x^3} - 4\left(x - \frac{1}{x}\right) = 0$

Put $t = x - \frac{1}{x}$

$\Rightarrow t^3 = x^3 - 3x^2 \frac{1}{x} + 3x\frac{1}{x^2} - \frac{1}{x^3} = x^3 - 3\left(x - \frac{1}{x}\right) - \frac{1}{x^3}$

$t^3 + 3t = x^3 - \frac{1}{x^3}$

Put in equation above $t^3 + 3t - 4t = 0 \Rightarrow t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$x - \frac{1}{x} = 0, \quad x - \frac{1}{x} = 1, \quad x - \frac{1}{x} = -1$

$x = \pm 1, \quad x^2 - x - 1 = 0, \quad x^2 + x - 1 = 0$

$x = \pm 1, \quad x = \frac{1 \pm \sqrt{5}}{2}, \quad x = \frac{-1 \pm \sqrt{5}}{2}$.

Q.25 (A) → (P); (B) → (S); (C) → (Q); (D) → (R)

(A) $x^2 - bx + c = 0 < \begin{matrix} \alpha \\ \beta \end{matrix}$

∵ $|\alpha - \beta| = 1$

$\Rightarrow (\alpha - \beta)^2 = 1$

$b^2 - 4c = 1$.

(B) Let α be common root then

$\alpha^2 + a\alpha + b = 0$

$\alpha^2 + b\alpha + a = 0$

$$\Rightarrow \frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b-a} = \frac{1}{b-a}$$

$$\Rightarrow \frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b-a} = \frac{1}{b-a}$$

$\Rightarrow \alpha = 1$ and $\alpha = -(a+b)$

$\therefore 1 = -(a+b)$.

(C) $\because \alpha + \beta = 1$ and $\alpha\beta = 3$

$\because \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= (1-6)^2 - 2(9)$

$= 25 - 18 = 7$

(D) $\because \Sigma\alpha = 7$

$\Sigma\alpha\beta = 16$

$\Sigma\alpha = 12$

$\therefore \Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)$

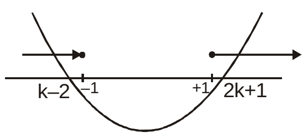
$= 49 - 32$

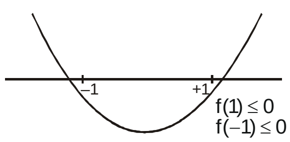
$\therefore \alpha^2 + \beta^2 + \gamma^2 = 17$

Q.26 (A)–Q,T; (B)–P,S; (C)–P,Q,S; (D)–P,S

(A) $x^2 - (3k-1)x + 2k^2 - 3k - 2 \geq 0$

$\Rightarrow [x - (k-2)][x - (2k+1)] \geq 0$





Hence $2k + 1 \geq 1$ & $k - 2 \leq -1$

i.e., $k \geq 0$ & $k \leq 1$

(B) $1 + a \geq 2\sqrt{a}$ {AM ≥ GM}

$1 + b \geq 2\sqrt{b}$; $1 + c \geq 2\sqrt{c}$; $1 + d \geq 2\sqrt{d}$

$\therefore (1+a)(1+b)(1+c)(1+d) \geq 16\sqrt{abcd} = 16$

$\therefore$ min. value = 16

(C) $5^{x+2} > 5^{\frac{-2}{x}} \Rightarrow x + 2 > \frac{-2}{x} \Rightarrow \frac{x^2 + 2x + 2}{x} > 0$

$$\Rightarrow \frac{(x+1)^2 + 1}{x} > 0 \Rightarrow x \in (0, \infty)$$

(D) $x^2 + 1 \geq 1$

$\therefore \log_3(3x^2 - x - 5) > \log_3(x^2 + 1)$

$\Rightarrow 3x^2 - x - 5 > x^2 + 1 \Rightarrow 2x^2 - x - 6 > 0$

$\Rightarrow (2x+3)(x-2) > 0$

$$\therefore x \in \left(-\infty, -\frac{3}{2}\right) \cup (2, \infty)$$

Q.27 (A) → (r), (B) → (p), (C) → (s), (D) → (p, q)

(A) $x^2 - 8x + k = 0$ with roots α and $\alpha + 4 = \beta$

$\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow 16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow k = 12$

(B) $\because (|x| - 2)(|x| - 3) = 0$

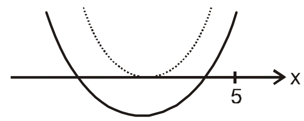
$\Rightarrow x = \pm 2$; $x = \pm 3$

$\therefore n = 4 \therefore \frac{n}{2} = 2$

(C) $\because b = (3 - i)(3 + i)$

$b = 10$

(D) $x^2 - 2kx + (k^2 + k - 5) = 0$



(i) $D \geq 0$

$\Rightarrow 4k^2 - 4(k^2 + k - 5) \geq 0$

$\Rightarrow k - 5 \leq 0$

(ii) $f(5) > 0$

$\Rightarrow 25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0 \Rightarrow (k-5)(k-4) > 0$

(iii) $-\frac{b}{2a} < 5 \Rightarrow k < 5$

$\Rightarrow k \in (-\infty, 4)$

So k may be 2, 3.

Q.28 (A)–R ; (B)–Q,R,S,T ; (C)–Q,R,S,T ; (D)–P,Q,R,S,T

(A) $(2^{(x^2+2)})^2 - 9 \cdot 2^{(x^2+2)} + 8 = 0$

$\Rightarrow y^2 - 9y + 8 = 0 \Rightarrow y = 8, 1$ (put $2^{(x^2+2)} = y$)

when $y = 8 \Rightarrow 2^{x^2+2} = 2^3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

when $y = 1 \Rightarrow 2^{x^2+2} = 2^0 \Rightarrow x^2 = -2$,

which is not possible.

Hence the equation has two solutions.

(B) Taking $x = \sin\theta = \dfrac{2z\tan\theta}{1-\tan^2\theta} = \dfrac{2z\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$

$\Rightarrow z = \dfrac{x(\cos^2\theta-\sin^2\theta)}{2\cos\theta}$ ....(1)

Similarly, $y = \dfrac{2z\sin\theta}{\cos^2\theta-\sin^2\theta}$

$\Rightarrow z = \dfrac{y(\cos^2\theta-\sin^2\theta)}{2\sin\theta}$ ....(2)

Compare (1) & (2), we get $\tan\theta = y/x$

$\therefore\ x\sin\theta = \dfrac{2xyz}{x^2-y^2}$

$\Rightarrow \sin\theta = \dfrac{2yz}{x^2-y^2}$ similarly $\cos\theta = \dfrac{2xz}{x^2-y^2}$

$\Rightarrow \dfrac{4z^2(x^2+y^2)}{(x^2-y^2)^2} = 1$

(C) We have $4x^2 - 16x + 15 < 0$

$\Rightarrow \dfrac{3}{2} < x < \dfrac{5}{2}$

$\therefore$ Integral solution of given inequality is x = 2.

Thus $\tan\alpha = 2$. It is given that $\cos\beta = \tan 45^\circ = 1$

$\therefore \sin(\alpha+\beta).\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$

$= \dfrac{1}{1+\cot^2\alpha} - (1-\cos^2\beta) = \dfrac{1}{1+\frac{1}{4}} - 0 = \dfrac{4}{5}$

(D) For real roots, discriminant ≥ 0

$\Rightarrow q^2 - 4p \geq 0 \Rightarrow q^2 \geq 4p$

For p = 1, $q^2 \geq 4 \Rightarrow q = 2, 3, 4$

p = 2, $q^2 \geq 8 \Rightarrow q = 3, 4$

p = 3, $q^2 \geq 12 \Rightarrow q = 4$

p = 4, $q^2 \geq 16 \Rightarrow q = 4$

Total seven solutions are possible

NUMERICAL VALUE BASED

Q.1 (2)

$(x^2+3x+2)(x^2+3x) = 120$

Let $x^2 + 3x = y$

$\Rightarrow y^2 + 2y - 120 = 0$

$\Rightarrow (y+12)(y-10) = 0$

$\Rightarrow y = -12 \Rightarrow x^2 + 3x + 12 = 0$

$\Rightarrow x \in \phi$

$y = 10 \Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow (x+5)(x-2) = 0$

$\Rightarrow x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

Q.2 (8)

$\left(5+2\sqrt{6}\right)^{x^2-3} + \dfrac{1}{\left(5+2\sqrt{6}\right)^{x^2-3}} = 10$

$\Rightarrow t + \dfrac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0$

$t = \dfrac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow (5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})$

or $\dfrac{1}{5+2\sqrt{6}}$

$\Rightarrow x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow x = 2$ or -2 or $-\sqrt{2}$ or $\sqrt{2}$

Product 8

Q.3 (11)

$2x^2 + 6x + a = 0$

Its roots are α, β

$\Rightarrow \alpha + \beta = -3$ & $\alpha\beta = \dfrac{a}{2}$

$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} < 2$

$\Rightarrow \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} < 2$

$\Rightarrow \dfrac{9-a}{a} < 1 \Rightarrow \dfrac{2a-9}{a} > 0$

$\Rightarrow a \in (-\infty, 0) \cup \left(\dfrac{9}{2}, \infty\right)$

$\Rightarrow 2a = 11$ is least prime.

Q.4 (1)

$x^2 + px + 1 = 0$ (roots a, b) $\quad a + b = -p,\ ab = 1$

; $x^2 + qx + 1 = 0$ (roots c, d) $\quad c + d = -q,\ cd = 1$

$a + b = -p,\ ab = 1 \Rightarrow c + d = -q,\ cd = 1$

RHS = $(a-c)(b-c)(a+d)(b+d) = (ab - ac - bc + c^2)(ab + ad + bd + d^2)$

$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$

$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$

$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2 - bd + c^2 + ac + bc + 1$

$[\because ab = cd = 1]$

$= c^2 + d^2 - a^2 - b^2 = (c+d)^2 - 2cd - (a+b)^2 + 2ab = q^2 - 2 - p^2 + 2 = q^2 - p^2$ = LHS. **Proved.**

2nd Method :
RHS = (ab – c(a + b) + c^2) (ab + d(ab + d(a + b) + d^2) = (c^2 + pc + 1) (1 – pd + d^2) ...(1)
Since c & d are the roots of the equation
$x^2 + qx + 1 = 0$
$\therefore$ $c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc$ & $d^2 + qd + 1 = 0$
$\Rightarrow d^2 + 1 = -qd$.
$\therefore$ (i) Becomes = (pc – qc) (–pd – qd) = c(p – q) (–d) (p + q) = –cd ($p^2 - q^2$)
= cd ($q^2 - p^2$) = $q^2 - p^2$ = LHS. **Proved.**

Q.5 **(73)**
$\because$ α, β are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$
$\therefore$ $\alpha + \beta = \frac{\lambda - 1}{\lambda}$ and $\alpha\beta = \frac{5}{\lambda}$
$\because$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4$ $\Rightarrow$
$\frac{\alpha^2 + \beta^2}{\alpha\beta} = 4$ $\Rightarrow$ $(\alpha + \beta)^2 = 6\alpha\beta$
$\Rightarrow$ $\frac{(\lambda - 1)^2}{\lambda^2} = \frac{30}{\lambda}$ $\Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ....(1)
$\because$ λ_1, λ_2 are roots of (1) $\therefore$ $\lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$
$\therefore$ $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{(\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2}$
$= \frac{(32)^2 - 2}{1} = 1022 \Rightarrow \left(\frac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{14}\right) = 73$

Q.6 (10)
α.β = b ; γδ = b – 2
$\Rightarrow$ αβγδ = b(b – 2) = 24
$\therefore$ $bx^2 + ax + 1 = 0$ has roots $\frac{1}{\alpha}, \frac{1}{\beta}$
$\Rightarrow$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-a}{b}$
$(b - 2)x^2 - ax + 1 = 0$ has root $\frac{1}{\gamma}, \frac{1}{\delta}$
$\Rightarrow$ $\frac{1}{\gamma} + \frac{1}{\delta} = \frac{a}{b - 2}$
$\frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{-a}{b} + \frac{a}{b - 2} = \frac{5}{6}$;
$\frac{+2a}{b(b - 2)} = \frac{5}{6}$; $\frac{+2a}{24} = \frac{5}{6}$
; a = 10.

Q.7 **(3)**
$x^2 + 2xy + 2y^2 + 4y + 7 = (x + y)^2 + (y + 2)^2 + 3 \geq 0 + 0 + 3$
$\therefore$ Least value = 3.

Q.8 (13)
$a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b\,(-9)$
$\Rightarrow$ a + b – 9 = 0 or a = b = – 9.
Which is rejected.
As a > b > – 9
$\Rightarrow$ a + b – 9 = 0 $\Rightarrow$ x = 1 is a root
other root = $\frac{-9}{a}$.
$\therefore$ $\alpha = \frac{-9}{a}$, β = 1
$\Rightarrow 4\beta - a\alpha = 4 - a\left(\frac{-9}{a}\right) = 4 + 9 = 13.$

Q.9 **(6)**
Let $t^2 - 2t + 2 = k$
$\Rightarrow$ $\alpha^2 - 6k\alpha - 2 = 0 \Rightarrow$ $\alpha^2 - 2 = 6k\alpha$
$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98} = \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k(\alpha^{99} - \beta^{99})$
$a_{100} - 2a_{98} = 6k.a_{99}$
$\frac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t - 1)^2 + 1]$
$\therefore$ min. value of $\frac{a_{100} - 2a_{98}}{a_{99}}$ is 6.

Q.10 **(11)**
Given that, roots of equation $x^2 - 10ax - 11b = 0$ are c, d
So c + d = 10a and cd = – 11b and a, b are the roots of equation $x^2 - 10cx - 11d = 0$
$\therefore$ a + b = 10c, ab = – 11d
So a + b + c + d = 10 (a + c) and
(c + d) – (a + b) = 10(a – c)
(c – a) – (b – d) + 10(c – a) = 0
$\Rightarrow$ b + d = 9(a + c) ....(i)
abcd = 121 bd
$\Rightarrow$ ac = 121 ....(ii)
b – d = 11(c – a) ....(iii)
c & a satisfies the equation $x^2 - 10ax - 11b = 0$ and $x^2 - 10cx - 11d = 0$ respectively
$\therefore$ $c^2 - 10ac - 11b = 0$
$a^2 - 10ca - 11d = 0$
$(c^2 - a^2) - 11(b - d) = 0$

(c – a) (c + a) = 11(b – d) = 11. 11 (c – a)
(by equation (iii))
c + a = 121
$\Rightarrow$ a + b + c + d = 10(c + a)
$\Rightarrow$ 10.121 $\Rightarrow$ $\frac{a + b + c + d}{110} = 11.$

KVPY
PREVIOUS YEAR'S

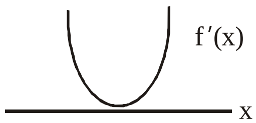
Q.1 (A)

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$D = 4a^2 - 4.3.b = 4(a^2-3b)$

If $a^2 < 2b \Rightarrow a^2 < 3b \Rightarrow D < 0 \Rightarrow f'(x) = 0$ has non real roots



Hence f (x) = 0 has 1 real and two imaginary roots

Q.2 (B)

$y = 2x^3 + ax + b$ $\qquad$ $y = 2x^3 + cx + d$

No solution

$2x^3 + ax + b \neq 2x^3 + cx + d$

$ax + b \neq cx + d$ for no real x

$(a - c)x \neq d - b$

$x \neq \frac{d-b}{a-c}$ $\qquad$ $a = c$

$(a - c)^2 + (b - d) = 0 + 6 - 1 = 5$

Q.3 (D)

$x^2 + 2ax + b^2 = 0$ $\qquad$ $x^2 + 2bx + c^2 = 0$

$D_1 > 0$ $\qquad$ $D_2 > 0$

$4a^2 + b^2 > 0$ $\qquad$ $4b^2 + 4c^2 > 0$

$a^2 > b^2$ .....(1) $\qquad$ $b^2 > c^2$ ......(2)

From (1) and (2)

$a^2 > b^2 > c^2 \Rightarrow a^2 > c^2 \Rightarrow c^2 - a^2 < 0$

$x^2 + 2cx + a^2 = 0$

$D = 4c^2 - 4a^2 < 0$ $\qquad$ No real roots

Q.4 (C)

say $f(x) = ax^2 + bx + c$

$f(x) = ax^2 + bx + c$

$f(2) = 4a + 2b + c = 10$

$f(-2) = 4a - 2b + c = -2$

$\Rightarrow 4b = 12$

$b = 3$

Q.5 (B)

$x + 2y + 4z = 9$;

$2xy + 8yz + 4xz = 26$;

$(x)(2y)(4z) = 24$

say roots of $P^3 - 9P^2 + 26P - 24 = 0$ are x, 2y and 4z

Here $(P - 2)(P^2 - 7P + 12) = 0$

$(P - 2)(P - 3)(P - 4) = 0$

now 4z = 4; 2y = 2 ; x = 3 then (3, 1, 1)

x = 2 ; 2y = 4 ; 4z = 3 then $\left(2, 2\frac{3}{2}\right)$

Q.6 (C)

$f(x) = x^3 + 3x^2 + 3x + 3 = 0$

f -(x) = 3x – 0

f(x) is Increasing function

Now $f(-3) = -6 < 02 + 6x + 3 = 3(x + 1)^2$

$f(-2) = 1 > 0$

$\therefore$ real root γ lies between – 3 and – 2

$\alpha + \beta + \gamma = -3$; $\quad -3 < \gamma < -2$

$\alpha + \beta - 3 < \alpha + \beta + \gamma < \alpha + \beta - 2$

$\alpha + \beta - 3 < -3 < \alpha + \beta - 2$

$-1 < \alpha + \beta < 0$

Q.7 (B)

$ax^2 + (a + b)x + b = 0$

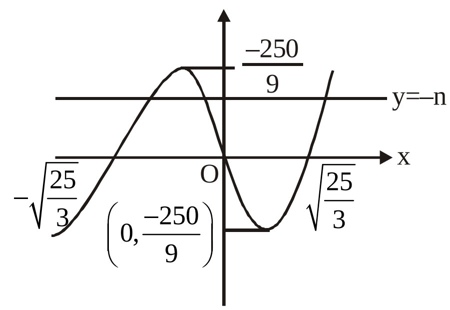
$(x + 1)(ax + b) = 0$ roots are $-1, \frac{-b}{a}$

Q.8 (C)

$x(3x^2 - 25) = -n$

$3x\left(x^2 - \frac{25}{3}\right) = -n$

$3x\left(x - \sqrt{\frac{25}{3}}\right)\left(x + \sqrt{\frac{25}{3}}\right) = -n$



$\therefore \quad n \in \left(-\frac{250}{9}, \frac{250}{9}\right)$

$\therefore$ But $n \in I$

Total integer = 55

Q.9 (C)

r is root, therefore $r^2 + 2r + 6 = 0$

$(r + 2)(r + 3)(r + 4)(r + 5) = (r^2 + 2r + 6 + 3r)$

$(r^2 + 4r + 5r + 20)$

$= (3r)(7r + 14)$

$= 21(r^2 + 2r) = 21(-6) = -126$

Q.10 (D)

$p(x) = x^2 - 5x + a$, $q(x) = x^2 - 3x + b$

{where, a, b $\in$ N}

hcf (p(x), q(x)) = x – 1

so x–1 is root of both p(x) & q (x)

$p(1) = 1 - 5 + a = 0 \Rightarrow a = 4$

$q(1) = 1 - 3 + b = 0 \Rightarrow b = 2$

$p(x) = x^2 - 5x + 4 = (x - 1)(x - 4)$

$p(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$

$k(x) = \ell cm\,(p(x), q(x)) = (x - 1)(x - 2)(x - 4)$

$(x - 1) + k(x) = (x - 1) + (x - 1)(x - 2)(x - 4)$

$= (x - 1)[1 + x^2 - 6x + 8]$

$= (x - 1)(x^2 - 6x + 9)$

$= (x - 1)(x - 3)^2$

Root 1, 3, 3

Sum of roots = 1 + 3 + 3 =7

Q.11 (B)

$a + b + c = 0$

$a = a^2 + b^2 + c^2$; $r = a^4 + b^4 + c^4$

$q^2 - 2r = (a^2 + b^2 + c^2)^2 - 2(a^4 + b^4 + c^4)$

$= 2a^2b^2 + 2b^2c^2 + 2a^2c^2 - a^4 - b^4 - c^4$

$= 2a^2c^2 + 2b^2c^2 - (a^2 - b^2)^2 - c^4$

$= 2c^2 (a^2 + b^2) - c^2(a - b)^2 - c^4$
$= c^2[2a^2 + 2b^2 - (a - b)^2 - c^2]$
$= c^2[2ab + a^2 + b^2 - c^2]$
$= c^2 [(a + b)^2 - c^2]$
$= 0$

Q.12 (C)

$$x + y = a; \frac{x^2}{x-1} + \frac{y^2}{y-1} = 4$$

$a \in [1, 2014]$

$$x + 1 + \frac{1}{x-1} + y + 1 + \frac{1}{y-1} = 4$$

$$(x-1) + \frac{1}{(x-1)} + (y-1) + \frac{1}{(y-1)} = 0$$

$$(a-2) + \frac{1}{(x-1)} + \frac{1}{(y-1)} = 0$$

$$(a-2) + \frac{(a-2)}{(x-1)(y-1)} = 0$$

$$(a-2)\left[1 + \frac{1}{xy + 1 - a}\right] = 0$$

$a \neq 2 [\text{for } a = 2 \text{ inf initely many solutions}]$

$xy + 1 - a + 1 = 0$
$x (a - x) - a + 2 = 0$

$\Rightarrow x^2 + ax - (2 - a) = 0$

$D = a^2 + 4(2 - a) = a^2 - 4a + 8$

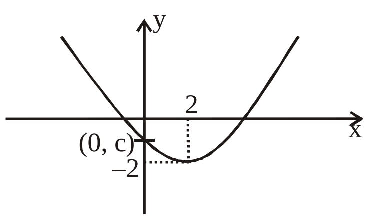
always + ve
there two real solutions.

$a \neq 2$ and $a \in [1, 2014]$

Total 2014 – 1 = 2013 values

Q.13 (B)

The graph according to the quation is



Clearly it can be observed
$c < 0$
$a > 0$

$$\frac{-b}{a} > 0 \Rightarrow -b > 0 \Rightarrow b < 0$$

$f (1) < 0 \Rightarrow a + b + c < 0$
$ab < 0$
$ac < 0$
$bc > 0$

Q.14 (C)

Let 'α' is the common root
So, $\alpha^2 + a\alpha + 2 = 0$ .....(i)
$\alpha^2 + 2\alpha + a = 0$ ....(ii)
$\Rightarrow (a - 2) \alpha + 2 - a = 0$
$\Rightarrow \alpha = 1$ is common root.
$\therefore 1^2 + a + 2 = 0 \Rightarrow a = -3.$
$f(x) + g(x) = 0$
$\Rightarrow 2x^2 + (a + 2) x + (a + 2) = 0$
$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow$ Sum of roots $= \frac{1}{2}$.

Q.15 (C)

$P(x) = ax^2 + bx + c = a(x- \alpha) (x- \beta)$
and $\alpha + \beta + \alpha\beta + 1 - 1 = (\alpha + 1)(\beta + 1) - 1$

$$= \frac{(a-b+c)}{a} - 1$$

$$\Rightarrow \alpha + \beta + \alpha\beta = \frac{b}{a} - 1 = \lambda_1 - 1$$

i.e., $\frac{b}{a}$ is integer $= \lambda_1$

If $b = a\lambda_1$
then, $c = a(2\lambda_1 - 1)$ {because a, b, c, are in A.P.}
$\therefore P(x) = ax^2 + a\lambda_1 x + a(2\lambda_1 - 1)$
$= a[x^2 + \lambda_1 x + (2\lambda_1 - 1)]$
$D = \lambda_1^2 - 4(2\lambda_1 - 1)$ is perfect square for intergral roots
$\Rightarrow D = \lambda_1^2 - 8\lambda_1 + 4$ is perfect square
Let $D = (\lambda_1 - 4)^2 - 12 = k^2$ {where $k \in 1$}
$\Rightarrow (\lambda_1 - 4 - k) (\lambda_1 - 4 + k) = 12$
This gives $\lambda_1 - 4 - k = 2$

$$\Rightarrow \frac{\& \lambda_1 - 4 + k = 6}{\lambda_1 - 4 \quad = 4 \,\&\, k = 1}$$

$\lambda_1 = 8$
$\therefore \alpha + \beta + \alpha\beta = 8 - 1 = 7$

Q.16 (B)

$t^2 = at + b$; $a, b \in I^+$
$t^3 = at^2 + bt$
$= a(at + b) + bt$
$= a^2t + bt + ab$
$\Rightarrow t^3 = (a^2 + b)t + ab$, check possibility for $a, b \in I^+$ from options.
(A) $a^2 + b = 4$
$ab = 3$ possible
(B) $a^2 + b = 8$
$ab = 5$ not possible
(C) $a^2 + b = 10$
$ab = 3$ possible
(D) $a^2 + b = 6$
$ab = 5$ possible

Q.17 (C)

$a^5 - a^3 - a = 2$; $a \in R^+$
Let $f(a) = a^5 - a^3 + a - 2$; { Note $f'(a) > 0 \ \forall a \in R$}
for $a^6 = 3 \Rightarrow a = 3^{1/6} = 1.2$ {use calculator}
we get $f(1.2) < 0$ and at $a = 4^{1/6} \Rightarrow f(4^{1/6}) > 0$
so one root in $a \in (3,4)$

Q.18 (B)

$D = 49n - 4n^2$
$= n(49 - 4n)$
$D \neq 0$ for any $n \in I^+$. So roots are distinct
For roots to be real $D \geq 0$

So $n \le \frac{49}{4}$
So n can be {1,2,3, ...12}
Clearly product of the roots is 1

Q.19 (C)

$\frac{a}{1} = \frac{1}{(a+10)}$

$a^2 + 10a - 1 = 0$
two value of a

Q.20 (A)

Given $|a| = \sqrt{4-\sqrt{5-a}}$
squaring
$a^2 = 4-\sqrt{5-a}$
$\Rightarrow a^4 + 16 - 8a^2 = 5 - a$
$\Rightarrow a^4 - 8a^2 + a + 11 = 0$
Similarly squaring other given equations & solving we can say that a, b, –c, –d are roots of $x^4 - 8x^2 + x + 11 = 0$
$\therefore$ product of roots
ab (–c) (–d) = 11
abcd = 11

Q.21 (B)

$x^4 - x^2 + 2x - 1 = 0$
$x^2 - (x-1)^2 = 0$
$\Rightarrow (x^2 + x - 1)(x^2 - x + 1) = 0$
$x^2 + x - 1 = 0$ has two real roots.

Q.22 (B)

Clearly statement 1 is false as they can have infinite solutions statements 2 is also false as $13^2 + 14^2 = 365$

Q.23 (C)

$S_m = \frac{m}{2}[2a + (m-1)d] = n$ .....(1)

$S_n = \frac{n}{2}[2a + (n-1)d] = m$ .....(2)

By (1) and (2)

$(m-n)a + (m-n)\{m+n-1\}\frac{d}{2} = -(m-n)$

$\Rightarrow 2a + (m+n-1)d = -2 (m \ne n)$

$\Rightarrow S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d] = -(m+n)$

Q.24 (B)

Product of digits of natural numebr will be a non negative integer
so, $n^2 - 10^n - 36 \ge 0$
$\Rightarrow n \in (-\infty, 5 - \sqrt{61}] \cup (5 + \sqrt{61}, \infty)$
but $n \in IN$
so $n \ge 13$; where $n \in N$
case - 1 for all 2 digit natural numbers max value of product of digits = 9 × 9 = 81
so $n^2 - 10n - 36 \le 81$
$\Rightarrow n \in [5 - \sqrt{142}, 5 + \sqrt{142},]$
but n is taken as a 2 digit natural no., so $13 \le n < 17$;
$\Rightarrow$ product of digits = 3, 4, 5 or 6 for 13, 14, 15 and 16 respectively checking n = 12
product of digits = 1× 3 = 3
and $13^2 - 10 \times 3 - 36 = 3$
so 13 satisfies the given conditon
Hence it is a solution
checking for n = 14
product = 1 × 4 = 4
142 – 10 × 14 – 36 = 196 – 140 – 36 = 20 > 6
and $n^2 - 10n - 36$ is increasing function for n>5; rest of the 2 digit integers won't satisfy the given condition
case-2 for all 3-digit integers max product = 9 × 9 × 9 = 729
The smallest 3 digit no. is 100
$f(n) = n^2 - 10n - 36$; $f(100) = 100^2 - 10 \times 100 - 36 = 8964 > 729$
and f(n) is increasing Hence no 3 digit Integers and similary any higher integer will not satisfy $\Rightarrow$ n =13 is the only answer.

Q.25 (B)

Q.26 **(C)**

Discriminant = $b^2 - \frac{8}{b} > 0$ (for real roots)

$= \frac{(b-2)(b^2+2b+4)}{b} > 4$

$\Rightarrow b \in (-\infty, 0) \cup (2, \infty)$

which is a subset of solution set of $b^2 - 3b + 2 > 0$
Hence (C).

Q.27 **(B)**

If α, β are roots of p(x) = 0
Roots of g(x) = 0 are $x^3 = \alpha, \beta$
Hence only 2 real roots

further $g(x) \ge b - \frac{a^2}{4} \forall x \in R$

Hence I & II are correct.

Q.28 **(A)**

Note that p(x) – 2x = a(x – 1) (x – 2) (x – 3) (x – 4)
since p(x) is a cubic polynomial, this is not possible.

Q.29 **(C)**

a + b = 5c and ab = –6d
c + d = 5a and cd = –6b
$\Rightarrow$ ac = 36
Now a satisfies first equation and c second one, so $a^2 - 5ac - 6d = 0$ & $c^2 - 5ac - 6b = 0$
adding these 2 are get
$(a + c)^2 - 12ac - 24(a + c) = 0$
$\Rightarrow (a + c)^2 - 24(a + c) - 12(36) = 0$
$\Rightarrow$ a + c = 36 or a + c = –12(a = c = –6)
hence a + c = 36 so b + d = 144

Q.30 (A)

$p_1(x)q_1(x) + p_2(x)q_2(x) = x^2 . 3x + 2$

$p_1(x) - p_2(x) = x^2 + (b_1 - b_2)x + (c_1 - c_2)$

$\Rightarrow q_1(x) = 1$ & $q_2(x) = .1$

$p_1(x) - p_2(x) = (x - 1)(x - 2)$

$p_1(x) = x^3 - 2020x^2 + b_1x + c_1$ < 1, 2, t

$t + 3 = 2020 \Rightarrow t = 2017$

$p_1(x) = (x - 1)(x - 2)(x - 2017)$

Similarly $p_2(x) = (x - 1)(x - 2)(x - 2018)$

(A) $p_1(3) + p_2(1) + 4028 = 0$

$p_1(3) = -4028$

$p_2(1) = 0$

Hence it is true

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (3)

$D < 0$

$(2(3k - 1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k - 4)(k - 2) < 0$

$2 < k < 4$

then $k = 3$

Q.2 (3)

If the root is $1 - 2i$, the other roots is $1 + 2i$

Sum = 2, Product = 5

$\therefore$ quadratic equation $z^2 - 2z + 5 = 0$

$\Rightarrow \alpha = -2, \beta = 5$

$\alpha - \beta = -2 - 5 = -7$

Q.3 (1)

$$E = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$$

Q.4 (1)

$2b = a + c$

$\frac{2a + 2}{3} = \frac{10}{3}$ and $\frac{2b + c}{3} = \frac{7}{3}$

$\Rightarrow a = 4$ $\left.\begin{array}{l} 2b + c = 7 \\ 2b - c = 4 \end{array}\right\}$ solving,

$b = \frac{11}{4}$ $c = \frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2 + \frac{11}{4}x + 1 = 0$

$\therefore$ The value of $(\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

Q.5 (3)

$(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Q.6 (324)

Quadratic Equation whose roots are α,β : $x^2-x-1=0$

$\therefore \alpha^2=\alpha+1 \Rightarrow \alpha^n=\alpha^{n-1}+\alpha^{n-2}$

$\beta^2=\beta+1 \Rightarrow \beta^n=\beta^{n-1}+\beta^{n-2}$

$\therefore P_n=P_{n-1}+P_{n-2}$

$\Rightarrow P_{n+1}=P_n+P_{n-1}$

$\Rightarrow 29=P_n+11 \Rightarrow P_n=18$

$\Rightarrow (Pn)^2=324$

Q.7 (1)

Q.8 (1)

Q.9 (1)

Q.10 (3)

Q.11 (1)

Q.12 (2)

Q.13 (3)

Q.14 (1)

Q.15 **(2)**

Q.16 **(18)**

Q.17 (66)

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (C)

$x^2 - 6x - 2 = 0$ having roots α and β $\Rightarrow \alpha^2 - 6\alpha - 2 = 0$

$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0 \Rightarrow$

$\alpha^{10} - 2\alpha^8 = 6\alpha^9$ .... (i)

similarly $\beta^{10} - 2\beta^8 = 6\beta^9$ .... (ii)

by (i) and (ii)

$(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9) \Rightarrow a_{10} - 2a_8 = 6a_9$

$\Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$

Aliter

$$\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^{10} - \beta^{10} + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^9(\alpha + \beta) - \beta^9(\alpha + \beta)}{2(\alpha^9 - \beta^9)} = \frac{\alpha + \beta}{2} = \frac{6}{2} = 3$$

Q.2 (B)

$$\begin{array}{l} x^2 + bx - 1 = 0 \\ x^2 + x + b = 0 \\ \hline \end{array}$$

$$\frac{x^2}{b^2+1} = \frac{x}{-1-b} = \frac{1}{1-b} \Rightarrow x = \frac{b^2+1}{-(b+1)} = \frac{-(b+1)}{1-b}$$

$\Rightarrow$ $(b^2+1)(1-b) = (b+1)^2$

$\Rightarrow$ $b^2 - b^3 + 1 - b = b^2 + 2b + 1$

$\Rightarrow$ $b^3 + 3b = 0 \Rightarrow b = 0$; $b^2 = -3 \Rightarrow b = 0, \pm\sqrt{3}\,i$

Q.3 (D)

p(x) will be of the form $ax^2 + c$. Since it has purely imaginary roots only.

Since p(x) is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.

Also $p(p(x)) = 0 \Rightarrow p(x)$ is purely imaginary

$\Rightarrow$ $ax^2 + c$ = purely imaginary

Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.

Thus .(D) is correct.

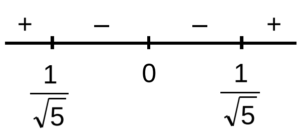
Q.4 (A, D)

$(x_1 + x_2)^2 - 4x_1x_2 < 1$

$$\frac{1}{\alpha^2} - 4 < 1$$

$$\Rightarrow 5 - \frac{1}{\alpha^2} > 0$$

$$\frac{5\alpha^2 - 1}{\alpha^2} > 0$$



$$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right) \quad ...(1)$$

D > 0

$1 - 4\alpha^2 > 0$

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right) \qquad ...(2)$$

(1) & (2)

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$$

Comprehension #1 (5 and 6)

Q.5 (D)

As α and β are roots of equation $x^2 - x - 1 = 0$, we get :

$\alpha^2 - \alpha - 1 = 0 \qquad \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \qquad \Rightarrow \beta^2 = \beta + 1$

$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12} = \alpha_{12}$

Q.6 (D)

$a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$, we get

$$a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$$

$$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0 \quad ....(i)$$

$$\text{and} \Rightarrow \frac{3p}{2} - \frac{3q}{2} = 0 \quad ....(ii)$$

$\Rightarrow$ p = q (from (ii))

$\Rightarrow$ 7p = 28 (from (i) and (ii))

$\Rightarrow$ p = 4 $\qquad \Rightarrow$ q = 4

$\Rightarrow$ p + 2q = 12

Q.7 (1,2,4)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta}$$

$$= \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1} \qquad \Rightarrow a_{r+2} - a_{r+1} = a_r$$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\dfrac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}}{(\alpha - \beta)} = \frac{10}{(10-\alpha)(10-\beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

Q.8 $x^2 + 20x - 2020 = 0$ has two roots $a,b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c,d \in$ complex

$ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c + d) + b^2(c + d) - c^2(a + b) - d^2(a + b)$

$= (c + d)(a^2 + b^2) - (a + b)(c^2 + d^2)$

$= (c + d)((a + b)^2 - 2ab) - (a + b)((c + d)^2 - 2cd)$

$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$

Q.9 (4)

$3x^2 + x - 1 = 4|x^2 - 1|$

If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4 \Rightarrow 7x^2 + x - 5 = 0$

say $f(x) = 7x^2 + x - 5$

$f(1) = 3$; $f(-1) = 1$; $f(0) = -1$

[Two Roots]

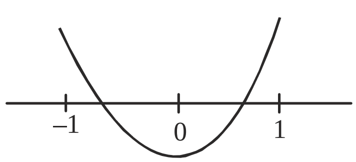
If $x \in (-\infty, -1] \cup [1, \infty)$

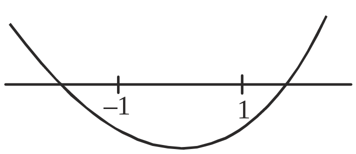
$3x^2 + x - 1 = 4x^2 - 4 \Rightarrow x^2 - x - 3 = 0$

Say $g(x) = x^2 - x - 3$

$g(-1) = -1$; $g(1) = -3$

[Two Roots]





So tatal 4 roots.